CS 477: Operational Program Semantics

Sasa Misailovic

Based on previous slides by Gul Agha, Elsa Gunter, Madhusudan Parthasarathy, Mahesh Viswanathan, and Vikram Adve

University of Illinois at Urbana-Champaign

Previously, on CS 477

Propositional Logic:

- Syntax
- Semantics
- Proof

(Homework/Quiz #1 is out: due next Thursday)

Simple Imperative Programming Language

- I $\in$ Identifiers
- N $\in$ Numerals
- B ::= true | false
| B & B | B or B | not B | E < E | E = E
- E::= N | I | E + E | E * E | E - E | - E
- S::= skip | S; S | I ::= E
| if B then S else S fi | while B do S od

Syntax -> Graphs

Reminder: Graph: (V, E)

- V is a set of vertices (nodes)
- $\mathrm{E} \subseteq V \times V$ is a relation denoting "connected" nodes. Elements $e \in E$ are edges: pairs of connected vertices $e = (v_1, v_2)$. Can be directed or undirected.

Common definitions:

- Post(v) – successor vertices of v, Pre(v) – direct predecessor vertices of v
- Path: a sequence of vertices s.t. $v_i \in Pre(v_{i+1})$. Cycle when the same vertex multiple times in the path, else simple. Length: number of vertices in a path.
- Acyclic graphs: no cycles.
- Tree: exists v_{root} (without predecessors) such that all other vertices reachable along unique paths
- Strongly connected component: all pairs of vertices mutually reachable
- Search: DFS, BFS; traversal: preorder, postorder, etc.

Syntax -> Graphs

- Parse Tree (from CS 374)

- Abstract Syntax Tree

- Control-flow Graph

Flow Graphs

- **Flow Graph:** A triple G=(N,A,s), where (N,A) is a (finite) directed graph, s ∈ N is a designated “initial” node, and there is a path from node s to every node n ∈ N.

- An *entry node* in a flow graph has no predecessors.
- An *exit node* in a flow graph has no successors.
- There is exactly one entry node, s. We can modify a general DAG to ensure this. *How?*
- We can also transform the graph to have only one exit node. *How?*

Control Flow Graph (CFG)

- **Flow Graph:** A triple $G=(N,A,s)$, where (N,A) is a (finite) directed graph, $s \in N$ is a designated "initial" node, and there is a path from node s to every node $n \in N$.

- **Control Flow Graph (CFG)** is a flow graph that represents all ***paths*** (sequences of statements) that might be traversed during program execution.
- Nodes in CFG are program statements, and edge (S_1,S_2) denotes that statement S_1 can be followed by S_2 in execution.
- In CFG, a node unreachable from s can be safely deleted. *Why?*
- Control flow graphs are usually *sparse*. I.e., $| A | = O(| N |)$. In fact, if only binary branching is allowed $| A | \leq 2 | N |$.

Control Flow Graph (CFG)

- **Basic Block** is a sequence of statements S_1 ... S_n such that execution control must reach S_1 before S_2, and, if S_1 is executed, then S_2 . . . S_n are all executed in that order
  - Unless some statement S_i causes the program to halt

- **Leader** is the first statement of a basic block

- **Maximal Basic Block** is a basic block with a maximum number of statements (n)

Control Flow Graph (CFG)

Let us refine our previous definition

- **CFG** is a directed graph in which:
- Each node is a single basic block
- There is an edge b1 → b2 if block b2 *may* be executed after block b1 in *some* execution

- We typically define it for a single procedure

- A CFG is a conservative approximation of the control flow! Why?

Example

LLVM bitcode (ver 3.9.1)

```
define i32 @fib(i32 %0) {
  %2 = icmp ult i32 %0, 2
  br i1 %2, label %12, label %3

; <label>:3:
  br label %4

; <label>:4:
  %5 = phi i32 [ %8, %4 ], [ 1, %3 ]
  %6 = phi i32 [ %5, %4 ], [ 0, %3 ]
  %7 = phi i32 [ %9, %4 ], [ 2, %3 ]
  %8 = add i32 %5, %6
  %9 = add i32 %7, 1
  %10 = icmp ugt i32 %9, %0
  br i1 %10, label %11, label %4

; <label>:11:
  br label %12

; <label>:12:
  %13 = phi i32 [%0, %1], [%8, %11]
  ret i32 %13
}
```

Source Code

```
unsigned fib(unsigned n) {
   int i;
   int f0 = 0, f1 = 1, f2;

   if (n <= 1) return n;

   for (i = 2; i <= n; i++) {
      f2 = f0 + f1;
      f0 = f1;
      f1 = f2;
   }
   return f2;
}
```

Dominance in Flow Graphs

- Let d, d1, d2, d3, n be nodes in G.
- d **dominates** n ("**d dom n**") *iff* every path from s to n contains d
- d **properly dominates** n if d dominates n and d $\neq$ n
- d **is the immediate dominator of** n ("**d idom n**") if d is the last proper dominator on any path from initial node to n,
- **DOM(**x**)** denotes the set of dominators of x,
- **Dominator tree:** the children of each node d are the nodes n such that "d idom n" (immediately dominates)

Dominator Properties

- **Lemma 1:** DOM(s) = { s }.
- **Lemma 2:** s dom d, for all nodes d in G.
- **Lemma 3:** The dominance relation on nodes in a flow graph is a ***partial ordering***
- ***Reflexive*** — *n dom n* is true for all n.
- ***Antisymmetric*** — If *d dom n*, then cannot be *n dom d*
- ***Transitive*** — $d1\ dom\ d2 \wedge d2\ dom\ d3 \Rightarrow d1\ dom\ d3$
- **Lemma 4:** The dominators of a node form a list.
- **Lemma 5:** Every node except s has a unique immediate dominator.

Postdominance

Def. Postdomination: node *p* postdominates a node *d* iff all paths to the exit node of the graph starting at *d* must go through *p*

Def. **Reverse Control Flow Graph** (RCFG) of a CFG has the same nodes as CFG and has edge $Y \rightarrow X$ if $X \rightarrow Y$ is an edge in CFG.

- p is a postdominator of d iff p dominates d in the RCFG.

Semantics

- Expresses the **meaning of syntax**

- Static semantics
  - Meaning based only on the form of the expression without executing it
  - Usually restricted to type checking / type inference

Dynamic semantics

- Method of **describing meaning of executing** a program
- Several different types:
  - Operational Semantics
  - Axiomatic Semantics
  - Denotational Semantics

- Different languages better suited to different types of semantics
- Different types of semantics serve different purposes

Operational Semantics

- Start with a simple notion of machine
- Describe how to execute (implement) programs of language on virtual machine, by describing how to execute each program statement (ie, following the *structure* of the program)
- Meaning of program is how its execution changes the state of the machine
- Useful as basis for implementations

Denotational Semantics

- Construct a function $\mathcal{M}$ assigning a mathematical meaning to each program construct
- Lambda calculus often used as the range of the meaning function
- Meaning function is compositional: meaning of construct built from meaning of parts
- Useful for proving properties of programs

Axiomatic Semantics

- Also called Floyd-Hoare Logic
- Based on formal logic (first order predicate calculus)
- Axiomatic Semantics is a logical system built from *axioms* and *inference rules*
- Mainly suited to simple imperative programming languages

Axiomatic Semantics

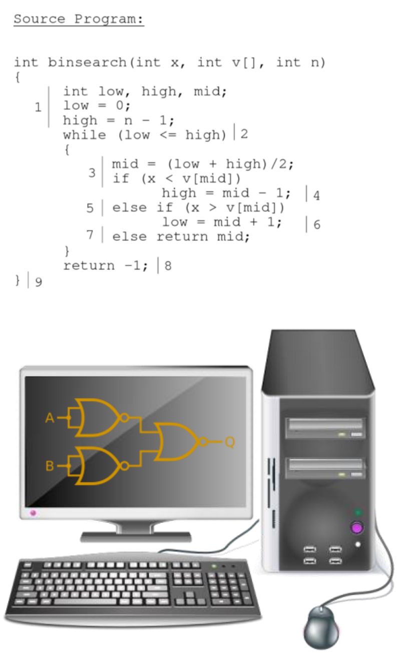
- Used to formally prove a property (*post-condition*) of the *state* (the values of the program variables) after the execution of program, assuming another property (*pre-condition*) of the state before execution

- Written :

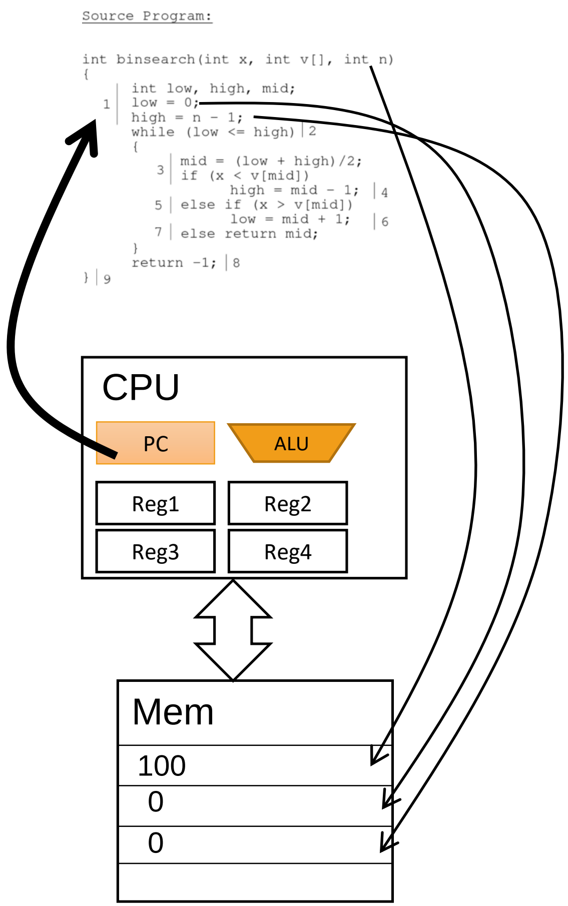
  {**Precondition**} ***Program*** {**Postcondition**}

Much more about it later in the course!

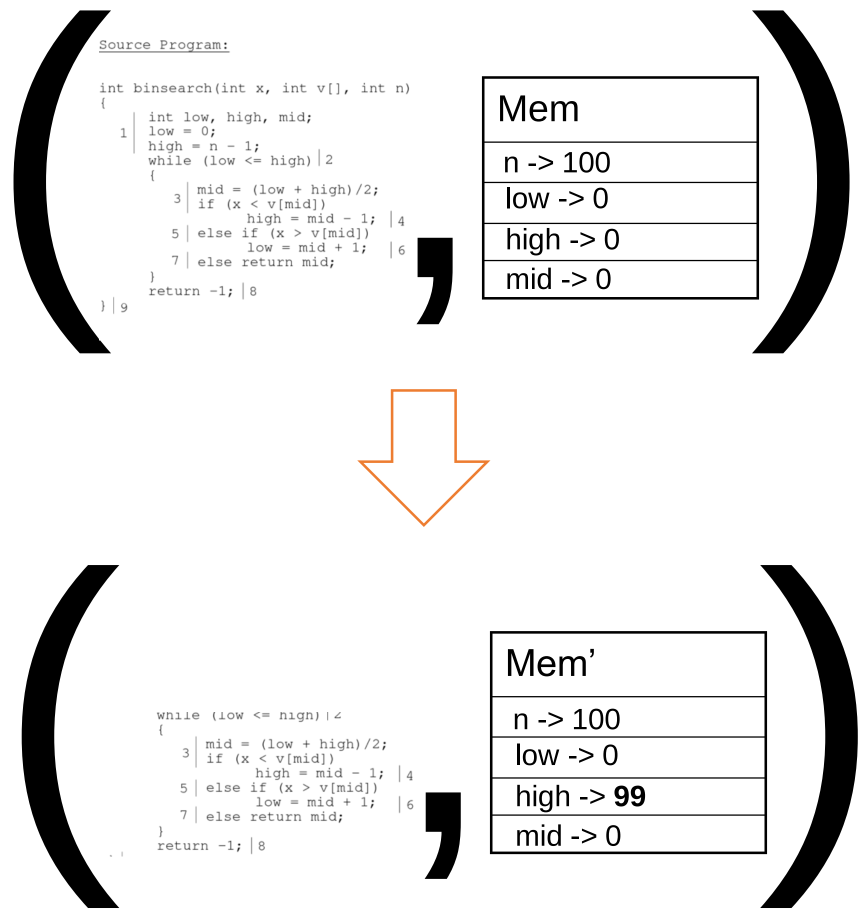
Modeling Program Environment



Real machine



Virtual machine



Mathematical Program Environment

Sources: https://www.researchgate.net/figure/Example-of-Control-Flow-Graph_fig5_4065402 and https://freesvg.org/computer-station-vector-graphics

Program Environment

Pair of code to execute + a valuation (aka state)

Code to execute: Next statement and program text that remains to be executed:
Statement_1; Other_Statements

A valuation of program variables:

- Mapping m: Identifiers-> Value

Program statements ("S_1; S_2; … S_n") transform the valuations. Execution is then:

- $m_2 = [[S_1]](m_1)$
- $m_3 = [[S_2]](m_2)$
- …
- $m_{n+1} = [[S_n]](m_n)$

- Also $(s_1, m_1) \rightarrow (s_2, m_2) \rightarrow (s_3, m_3) \rightarrow \ \ldots \ \longrightarrow (s_n, m_n) \longrightarrow (\cdot, m_{n+1})$.
  We can define the sequence $(s_1, m_1), (s_2, m_2), (s_3, m_3), \ldots, (s_n, m_n), (\cdot, m_{n+1})$ or its projection $(m_1, \ldots m_n)$ as the **trace** of execution

Natural Semantics (“Big-step Semantics”)

- Aka Structural Operational Semantics, aka “Big Step Semantics”

- Provide value for a program by rules and derivations, similar to type derivations

- Rule conclusions look like

$$(C, m) \Downarrow m'$$

“Evaluating a command C in the state m results in the new state m’ ”

or

$$(E, m) \Downarrow v$$

“Evaluating an expression E in the state m results in the value v’”

Simple Imperative Programming Language

- I $\in$ Identifiers
- N $\in$ Numerals
- B ::= true | false
  | B & B | B or B | not B | E < E | E = E
- E::= N | I | E + E | E * E | E - E | - E
- C::= skip | C;C | I ::= E
  | if B then C else C fi | while B do C od

Natural Semantics of Atomic Expressions

- **Identifiers:** $(k,m) \Downarrow m(k)$

- **Numerals are values:** $(N,m) \Downarrow N$

- **Booleans:** $(\text{true},m) \Downarrow \text{true}$

  $(\text{false} ,m) \Downarrow \text{false}$

Booleans:

$$\frac{(B, m) \Downarrow \text{false}}{(B \mathbin{\&} B', m) \Downarrow \text{false}} \qquad \frac{(B, m) \Downarrow \text{true} \qquad (B', m) \Downarrow b}{(B \mathbin{\&} B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(B \text{ or } B', m) \Downarrow \text{true}} \qquad \frac{(B, m) \Downarrow \text{false} \quad (B', m) \Downarrow b}{(B \text{ or } B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(\text{not } B, m) \Downarrow \text{false}} \qquad \frac{(B, m) \Downarrow \text{false}}{(\text{not } B, m) \Downarrow \text{true}}$$

Binary Relations

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \text{ rop } V = b}{(E \text{ rop } E', m) \Downarrow b}$$

- By U rop V = b, we mean does (the meaning of) the relation rop hold on the meaning of U and V

- May be specified by a mathematical expression/equation or rules matching U and V

Arithmetic Expressions

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \text{ op } V = N}{(E \text{ op } E', m) \Downarrow N}$$

where N is the specified value for (mathematical) U op V

Commands

Skip: $(\text{skip}, m) \Downarrow m$

Assignment:

$$\frac{(E,m) \Downarrow V}{(k := E,m) \Downarrow m\ [k \text{ <-- } V\ ]}$$

Sequencing:

$$\frac{(C,m) \Downarrow m' \quad (C',m') \Downarrow m''}{(C;\ C',\ m) \Downarrow m''}$$

If Then Else Command

$$\frac{(B,m) \Downarrow \text{true} \quad (C,m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$

$$\frac{(B,m) \Downarrow \text{false} \quad (C',m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$

Example: If Then Else Rule

$$\frac{}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \{x \text{ -> } 7\}) \Downarrow \,?}$$

Example: If Then Else Rule

$$\frac{\overline{\quad (x > 5, \{x \text{ -> } 7\}) \Downarrow ? \quad}}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \ \{x \text{ -> } 7\})}$$

$$\Downarrow \ ?$$

Example: Arith Relation

$$\dfrac{\dfrac{? > ? = ? \qquad (x,\{x\text{->}7\}) \Downarrow ? \quad (5,\{x\text{->}7\}) \Downarrow ?}{(x > 5, \{x \text{->} 7\}) \Downarrow ?}}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \{x \text{->} 7\}) \Downarrow ?}$$

Example: Identifier(s)

$$\dfrac{\dfrac{\begin{array}{c} 7 > 5 = \text{true} \\ (x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5 \end{array}}{(x > 5, \{x \text{ -> } 7\})\Downarrow ?}}{(\text{if } x > 5 \text{ then } y\text{:= } 2 + 3 \text{ else } y\text{:=}3 + 4 \text{ fi}, \{x \text{ -> } 7\}) \Downarrow ?}$$

Example: Arith Relation

$$\frac{\dfrac{\begin{array}{c}7 > 5 = \text{true}\\ (x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5\end{array}}{(x > 5,\ \{x \text{ -> } 7\})\Downarrow \text{true}}}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi},\ \{x \text{ -> } 7\})}$$

$$\Downarrow ?$$

Example: If Then Else Rule

$$\dfrac{\dfrac{\begin{array}{c} 7 > 5 = \text{true} \\ (x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5 \end{array}}{(x > 5,\ \{x \text{ -> } 7\})\Downarrow \text{true}} \qquad \dfrac{}{\begin{array}{c}(y{:=}\ 2 + 3,\ \{x\text{-> } 7\} \\ \Downarrow\ ?\end{array}}}{\begin{array}{c}(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi},\ \{x \text{ -> } 7\}) \\ \Downarrow\ ?\end{array}}$$

Example: Assignment

$$\dfrac{\dfrac{\begin{array}{c} 7 > 5 = \text{true} \\ (x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5 \end{array}}{(x > 5, \{x \text{->} 7\})\Downarrow \text{true}} \qquad \dfrac{\dfrac{}{(2+3, \{x\text{->}7\})\Downarrow ?}}{(y{:=}\ 2 + 3, \{x\text{->}\ 7\} \ \Downarrow\ ?}}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \{x \text{->} 7\}) \ \Downarrow\ ?}$$

Example: Arith Op

$$
\dfrac{
\dfrac{7 > 5 = \text{true} \qquad (x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5}{(x > 5,\ \{x \text{->} 7\})\Downarrow \text{true}}
\qquad
\dfrac{\dfrac{? + ? = ? \qquad (2,\{x\text{->}7\})\Downarrow ? \quad (3,\{x\text{->}7\})\Downarrow ?}{(2+3,\ \{x\text{->}7\})\Downarrow ?}}{(y{:=}\ 2 + 3,\ \{x\text{->}\ 7\}\ \Downarrow ?\qquad .}
}{
(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi},\ \{x \text{->} 7\}) \Downarrow ?
}
$$

Example: Numerals

$$
\dfrac{
\dfrac{7 > 5 = \text{true} \qquad (x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5}{(x > 5,\ \{x \text{ -> } 7\})\Downarrow \text{true}}
\qquad
\dfrac{\dfrac{\begin{array}{c} 2 + 3 = 5 \\ (2,\{x\text{->}7\})\Downarrow 2 \quad (3,\{x\text{->}7\})\ \Downarrow 3 \end{array}}{(2+3,\ \{x\text{->}7\})\Downarrow ?}}{\begin{array}{l}(y{:=}\ 2 + 3,\ \{x\text{-> } 7\} \\ \Downarrow\ ? \end{array}}
}{\begin{array}{c}(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi},\ \{x \text{ -> } 7\}) \\ \Downarrow\ ? \end{array}}
$$

Example: Arith Op

$$
\dfrac{
\dfrac{\begin{array}{c} 7 > 5 = \text{true} \\ (x,\{x->7\})\Downarrow 7 \quad (5,\{x->7\})\Downarrow 5 \end{array}}{(x > 5, \{x -> 7\})\Downarrow \text{true}}
\qquad
\dfrac{\dfrac{\begin{array}{c} 2 + 3 = 5 \\ (2,\{x->7\})\Downarrow 2 \quad (3,\{x->7\})\Downarrow 3 \end{array}}{(2+3, \{x->7\})\Downarrow 5}}{\begin{array}{l} (y:= 2 + 3, \{x-> 7\} \\ \Downarrow ? \end{array}}
}{\begin{array}{c} (\text{if } x > 5 \text{ then } y:= 2 + 3 \text{ else } y:=3 + 4 \text{ fi}, \ \{x -> 7\}) \\ \Downarrow \ ? \end{array}}
$$

Example: Assignment

$$\dfrac{\dfrac{\begin{array}{c}7 > 5 = \text{true}\\ (x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5\end{array}}{(x > 5,\ \{x \text{ -> } 7\})\Downarrow \text{true}} \qquad \dfrac{\dfrac{\begin{array}{c}2 + 3 = 5\\ (2,\{x\text{->}7\})\Downarrow 2 \quad (3,\{x\text{->}7\})\ \Downarrow 3\end{array}}{(2{+}3,\ \{x\text{->}7\})\Downarrow 5}}{\begin{array}{c}(y{:=}\ 2 + 3,\ \{x\text{-> } 7\}\\ \Downarrow\ \{x\text{->}7,\ y\text{->}5\}\end{array}}}{\begin{array}{c}(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi},\ \{x \text{ -> } 7\})\\ \Downarrow ?\end{array}}$$

Example: If Then Else Rule

$$
\dfrac{
\dfrac{\begin{array}{c} 7 > 5 = \text{true} \\ (x,\{x->7\}) \Downarrow 7 \quad (5,\{x->7\}) \Downarrow 5 \end{array}}{(x > 5, \{x -> 7\}) \Downarrow \text{true}}
\qquad
\dfrac{\dfrac{\begin{array}{c} 2 + 3 = 5 \\ (2,\{x->7\}) \Downarrow 2 \quad (3,\{x->7\}) \Downarrow 3 \end{array}}{(2+3, \{x->7\}) \Downarrow 5}}{\begin{array}{c}(y := 2 + 3, \{x-> 7\} \\ \Downarrow \{x->7, y->5\}\end{array}}
}{
\begin{array}{c}(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi}, \{x -> 7\}) \\ \Downarrow \mathbf{\{x->7, y->5\}}\end{array}
}
$$

While Command

$$\frac{(B,\ m) \Downarrow \text{false}}{(\text{while } B \text{ do } C \text{ od},\ m) \Downarrow m}$$

$$\frac{\overset{1}{(B,m) \Downarrow \text{true}} \quad \overset{2}{(C,m) \Downarrow m'} \quad \overset{3}{(\text{while } B \text{ do } C \text{ od},\ m') \Downarrow m''}}{(\text{while } B \text{ do } C \text{ od},\ m) \Downarrow m''}$$

Example: While Rule

$$\dfrac{\text{1} \quad (x > 5, \{x\text{->}7\}) \Downarrow \text{true} \qquad \text{2} \quad (x := x\text{-}5, \{x\text{->}7\}) \Downarrow \{x\text{->}2\} \qquad \text{3} \quad \dfrac{(x > 5, \{x\text{->}2\}) \Downarrow \text{false}}{\text{while } x > 5 \text{ do } x := x\text{-}5 \text{ od; } \{x \text{ -> } 2\}) \Downarrow \{x\text{->}2\}}}{(\text{while } x > 5 \text{ do } x := x\text{-}5 \text{ od}, \{x \text{ -> } 7\}) \Downarrow \{x\text{->}2\}}$$

While Command

$$\frac{(B,\ m) \Downarrow \text{false}}{(\text{while } B \text{ do } C \text{ od},\ m) \Downarrow m}$$

$$\frac{(B,m) \Downarrow \text{true} \quad (C,m) \Downarrow m' \quad (\text{while } B \text{ do } C \text{ od},\ m') \Downarrow m''}{(\text{while } B \text{ do } C \text{ od},\ m) \Downarrow m''}$$

The rule assumes the loop terminates!

While Command

$$\frac{(B, m) \Downarrow \text{false}}{(\text{while B do C od}, m) \Downarrow m}$$

$$\frac{(B,m) \Downarrow \text{true} \quad (C,m) \Downarrow m' \quad (\text{while B do C od}, m') \Downarrow m''}{(\text{while B do C od}, m) \Downarrow m''}$$

The rule assumes the loop terminates!

$$\frac{\textbf{\textit{? ? ?}}}{\text{while (x>0) do x:=x+1od, \{x->1\}} \Downarrow \textbf{? ? ?}}$$

Interpretation Versus Compilation

- A **compiler** from language L1 to language L2 is a program that takes an L1 program and for each piece of code in L1 generates a piece of code in L2 of same meaning
- An **interpreter** of L1 in L2 is an L2 program that executes the meaning of a given L1 program
- Compiler would examine the body of a loop once; an interpreter would examine it every time the loop was executed

Interpreter

- An *Interpreter* represents the operational semantics of a language L1 (source language) in the language of implementation L2 (target language)
- Built incrementally
  - Start with literals
  - Variables
  - Primitive operations
  - Evaluation of expressions
  - Evaluation of commands/declarations

Interpreter

- Takes abstract syntax trees as input
  - In simple cases could be just strings
- One procedure for each syntactic category (nonterminal)
  - eg one for expressions, another for commands
- If Natural semantics used, tells how to compute final value from code
- If Transition semantics used, tells how to compute next "state"
  - To get final value, put in a loop

Natural Semantics Interpreter Implementation

- Identifiers: (k,m) $\Downarrow$ m(k)
- Numerals are values: (N,m) $\Downarrow$ N

- Conditionals: $\dfrac{(B,m) \Downarrow \text{true} \quad (C,m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$ $\quad$ $\dfrac{(B,m) \Downarrow \text{false} \quad (C',m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$

```
compute_exp (Var(v), m) = look_up v m
compute_exp (Int(n), _) = Num (n)
…
compute_com (IfExp(b,c1,c2), m) =
       if compute_exp (b,m) = Bool(true)
       then compute_com (c1,m)
       else compute_com (c2,m)
```

Natural Semantics Interpreter Implementation

- Loop: $$\frac{(B, m) \Downarrow \text{false}}{(\text{while } B \text{ do } C \text{ od}, m) \Downarrow m} \qquad \frac{(B,m) \Downarrow \text{true} \quad (C,m) \Downarrow m' \quad (\text{while } B \text{ do } C \text{ od}, m') \Downarrow m''}{(\text{while } B \text{ do C od}, m) \Downarrow m''}$$

```
compute_com (While(b,c), m) =
      if compute_exp (b,m) = Bool(false)
      then m
      else compute_com
              (While(b,c), compute_com(c,m))
```

- May fail to terminate - exceed stack limits
  - Returns no useful information then