CS 526

Advanced

Compiler

Construction

http://misailo.cs.Illinois.edu/courses/cs526

STATIC SINGLE ASSIGNMENT

The slides adapted from Vikram Adve

References

Cytron, Ferrante, Rosen, Wegman, and Zadeck,
"Efficiently Computing Static Single Assignment Form and the Control Dependence Graph,"
ACM Trans. on Programming Languages and Systems, 13(4), Oct. 1991, pp. 451–490.

Muchnick, Section 8.11 (*partially covered*).

Engineering a Compiler, Section 5.4.2 (*partially covered*).

Definition of φ Function

In a basic block B with N predecessors, P1, P2, . . . , PN,

$$\mathbf{X = \varphi(V1, V2, \ldots, VN)}$$

assigns X = Vi if control enters block B from Pi , $1 \leq i \leq N$

Properties of φ-functions:

- φ is not an executable operation.
- φ has exactly as many arguments as the number of incoming basic block edges
- Think about the argument Vi as being evaluated on CFG edge from predecessor Pi to B

Which Variables to Convert?

Convert all variables to SSA form, ***except*** . . .

Arrays: Array elements do not have an explicit name (although note ArraySSA)

Variables that may have aliases: do not have a unique name

Volatile variables: can be modified "unexpectedly"

E.g., In LLVM, only scalar variables in virtual registers are in SSA form.

LLVM: Mem2reg

-mem2reg: Promote Memory to Register

“This file promotes memory references to be register references. It promotes alloca instructions which only have loads and stores as uses. An alloca is transformed by using dominator frontiers to place phi nodes, then traversing the function in depth-first order to rewrite loads and stores as appropriate. This is just the standard SSA construction algorithm to construct “pruned” SSA form.”

```
int f(int x) {
  int y = x + 1;
  return y
}
```

No optimizations

```
...
%6 = alloca i32, align 4
%7 = load i32, i32* %0, align 4
%8 = add nsw i32 1, %7
store i32 %8, i32* %6, align 4
%9 = load i32, i32* %6, align 4
ret i32 %9
```

After mem2reg

```
%3 = add nsw i32 %0, 1
ret i32 %3
```

More Definitions

Use of a variable: A use of variable X is a reference that may read the value stored in the location named X.

Definition of a variable: A definition (def) of a variable X is a reference that may store a value into the location named X. *Examples*: Assignment; input I/O

Ambiguity of definitions:

Unambiguous definition **(must)**: guaranteed to store to X

Ambiguous definition **(may)**: may store to X

Q. Where does ambiguity come from?

We define ambiguous/unambiguous use similarly.

Def-Use Chains

- **Def-use chain**: The set of uses reached by a particular definition.

- **Use-def chain**: The set of definitions reaching a particular use

Definition of SSA Form

A program is in SSA form if:

- each variable is assigned a value in **exactly one** statement
- each **use** of a variable is **dominated** by the **definition**

Advantages of SSA Form

Makes def-use and use-def chains explicit:

These chains are foundation of many dataflow optimizations

- We will see some soon!

Compact, flow-sensitive* def-use information

- fewer def-use edges per variable: one per CFG edge

* Takes the order of statements into account

Advantages of SSA Form (cont.)

No anti- and output dependences on SSA variables

- Direct dependence: **A=1; B=A+1**
- Antidependence: A=1; **B=A+1;A=2**
- Output dependence: **A=1;A=2;** B=A+1

$\left.\right\}$ **Cannot reoder**

Explicit merging of values (φ): key additional information

Can serve as **IR for code transformations** (see LLVM)

Constructing SSA Form

Simple algorithm

1. insert φ-functions for every variable at every join
2. solve reaching definitions
3. rename each use to the def that reaches it (unique)

What's wrong with this approach?

1. too many φ-functions (precision)
2. too many φ-functions (space)
3. too many φ-functions (time)

Where do we place φ-functions?

```
V=...; U=...; W=...;
if (...) then {
     V = ...;
     if (...) {
        U = V + 1;
     } else {
        U = V + 2;
     }

     W = U + 1;
}
```

- For V?
- For U?
- For W?

Where do we place φ-functions?

```
V0=...; U0=...; W0=...;
if (...) then {
      V1 = ...;
      if (...) {
          U1 = V1 + 1;
      } else {
          U2 = V1 + 2;
      }
      V2=φ(V1, V1);U3=φ(U1, U2);W1=φ(W0, W0)
      W1 = U3 + 1;
}
V3=φ(V0, V1); U4=φ(U0, U3); W2=φ(W0, W1)
```

- For V?
- For U?
- For W?

Intuition for SSA Construction

Informal Conditions

If a block X contains an assignment to a variable V, then a φ-function must be inserted in each block Z such that:

1. there is a non-empty path between X and Z,
2. there is a path from the entry block (s) to Z that does not go through X,
3. Z is the first node on the path from X that satisfies point 2.

Intuition for SSA Construction

Informal Conditions

If block X contains an assignment to a variable V, then a φ-function must be inserted in each block Z such that:

1. there is a non-empty path between X and Z, and

   the value of V computed in X reaches Z

2. there is a path from the entry block (s) to Z that does not go through X

   there is a path that does not go through X, so some other value of V reaches Z along that path(ignore bugs due to uses of uninitialized variables). So, two values must be merged at X with a φ

3. Z is the first node on the path from X to Z that satisfies point 2

   the φ for the value coming from X is placed in Z and not in some earlier node on the path

Intuition for SSA Construction

Informal Conditions

Iterating the Placement Conditions:

- After a φ is inserted at Z, the above process must be repeated for Z because the φ is effectively a new definition of V.
- For each block X and variable V, there must be at most one φ for V in X.

This means that the above iterative process can be done with a single worklist of nodes for each variable V, initialized to handle all original assignment nodes X simultaneously.

Minimal SSA

A program is in SSA form if:

- each variable is assigned a value in **exactly one** statement
- each **use** of a variable is **dominated** by the **definition** i.e., the use can refer to a unique name.

Minimal SSA: As few as possible φ-functions,

Pruned SSA: As few as possible φ-functions and no dead φ-functions (i.e., the defined variable is used later)

- One needs to compute liveness information
- More precise, but requires additional time

SSA Construction Algorithm

Steps:

1. Compute the dominance frontiers*
2. Insert φ-functions
3. Rename the variables

Thm. Any program can be put into minimal SSA form using the previous algorithm. [Refer to the paper for proof]

Dominance in Flow Graphs (review)

Let d, d1, d2, d3, n be nodes in G.

d **dominates** n ("d dom n") iff every path in G from s to n contains d

d **properly dominates n** ("d pdom n") if d dominates n and d $\neq$ n

d **is the immediate dominator of** n ("d idom n")
if d is the last proper dominator on any path from initial node to n,

DOM(x**)** denotes the set of dominators of x,

Dominator tree*: the children of each node d are the nodes n such that "d idom n" (d immediately dominates n)

Dominance Frontier

The dominance frontier of node X is the **set of nodes Y** such that **X dominates a predecessor of Y**, but X does not <u>properly</u> dominate Y *

DF(X) = {Y | ∃ P ∈ Pred(Y) : X dom P and not (X pdom Y)}

We can split DF(X) in two groups of sets:

$DF_{local}(X) \equiv \{Y \in Succ(X) \mid \text{not } X \text{ idom } Y\}$

$DF_{up}(Z) \equiv \{Y \in DF(Z) \mid \exists W.\ W \text{ idom } Z \text{ and not } W \text{ pdom } Y\}$

Then:

$$DF(X) = DF_{local}(X) \cup \bigcup_{Z \in Children(X)} DF_{up}(Z)$$

* **child, parent, ancestor, and descendant always refer to the dominator tree. predecessor, successor, and path always refer to CFG**

Dominance Frontier Algorithm

for each X in a bottom-up traversal of the dominator tree (visit the node X in the tree after visiting its children):

```
    DF(X) ← Ø

    for each Y ∈ succ(X) /* local */
        if not X idom Y then
            DF(X) ← DF(X) U {Y}

    for each Z ∈ children(X) /* up */
        for each Y ∈ DF(Z)
            if not X idom Y then
                DF(X) ← DF(X) U {Y}
```

Dominance and LLVM

LLVM mainline

Main Page | Related Pages | Modules | Namespaces | Classes | Files

File List | File Members

Dominators.h

Go to the documentation of this file.

```
//===- Dominators.h - Dominator Info Calculation ----------------*- C++ -*-===//
//
//                     The LLVM Compiler Infrastructure
//
// This file is distributed under the University of Illinois Open Source
// License. See LICENSE.TXT for details.
//
//===----------------------------------------------------------------------===//
//
// This file defines the DominatorTree class, which provides fast and efficient
// dominance queries.
//
//===----------------------------------------------------------------------===//

```

DominanceFrontier.h

Go to the documentation of this file.

```
//===- llvm/Analysis/DominanceFrontier.h - Dominator Frontiers --*- C++ -*-===//
//
//                     The LLVM Compiler Infrastructure
//
// This file is distributed under the University of Illinois Open Source
// License. See LICENSE.TXT for details.
//
//===----------------------------------------------------------------------===//
//
// This file defines the DominanceFrontier class, which calculate and holds the
// dominance frontier for a function.
//
// This should be considered deprecated, don't add any more uses of this data
// structure.
//
//===----------------------------------------------------------------------===//

#ifndef LLVM_ANALYSIS_DOMINANCEFRONTIER_H
#define LLVM_ANALYSIS_DOMINANCEFRONTIER_H

#include "llvm/IR/Dominators.h"
#include <map>
#include <set>

namespace llvm {

//===----------------------------------------------------------------------===//
/// DominanceFrontierBase - Common base class for computing forward and inverse
/// dominance frontiers for a function.
///
template <class BlockT>
class DominanceFrontierBase {
public:
  typedef std::set<BlockT *> DomSetType;                // Dom set for a bb
  typedef std::map<BlockT *, DomSetType> DomSetMapType; // Dom set map

protected:
  typedef GraphTraits<BlockT *> BlockTraits;

```

SSA Construction Algorithm

Steps:

1. Compute the dominance frontiers
2. Insert φ-functions
3. Rename the variables

Insert φ-functions

```
for each variable V
    HasAlready ← ∅
    EverOnWorkList ← ∅
    WorkList ← ∅
    for each node X that may modify V
        EverOnWorkList ← EverOnWorkList ⋃ {X}
        WorkList ← WorkList ⋃ {X}
```

Insert φ-functions

```
for each variable V
    HasAlready ← ∅
    EverOnWorkList ← ∅
    WorkList ← ∅
    for each node X that may modify V
        EverOnWorkList ← EverOnWorkList ⋃ {X}
        WorkList ← WorkList ⋃ {X}

    while WorkList ≠ ∅
        remove X from WorkList
        for each Y ∈ DF(X)
            if Y ∉ HasAlready then
                insert a ϕ-node for V at Y
                HasAlready ← HasAlready ⋃ {Y}
                if Y ∉ EverOnWorkList then
                    EverOnWorkList ← EverOnWorkList ⋃ {Y}
                    WorkList ← WorkList ⋃ {Y}
```

Renaming Variables*

Renaming definitions is easy – just keep the counter for each variable.

To **rename each use** of V :

(a) **Use in non-φ-functions**: Refer to immediately dominating definition of V (+ φ nodes inserted for V).

preorder on Dominator Tree!

(b) **Use as a φ-function operand**: Refer to the definition that immediately dominates the node with the incoming CFG edge (not the node with the φ-function)

rename the φ-operand when processing the predecessor basic block!

* For the full algorithm refer to the paper

```
j=1;

while (j < X)
    ++j;

N = j;
```

```
A: j = 1;

B: if (j >= X) goto E;

S:
j = j+1;
if (j < X) goto S;

E:
N = j;
```

```
j=1;

while (j < X)
    ++j;

N = j;
```

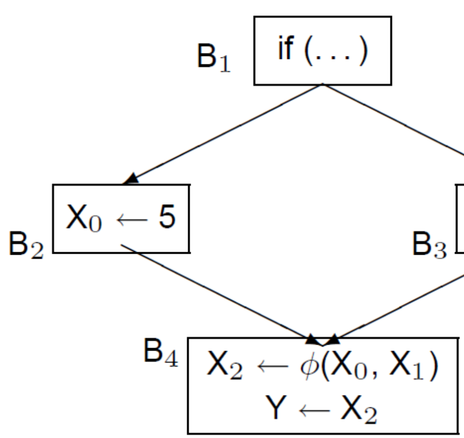
```
A: j0 = 1;

B: if (j0 >= X) goto E;

S: j1 = φ(j0, j2)
j2 = j1+1;
if (j2 < X) goto S;

E: j3 = φ(j0, j2)
N = j3;
```

Translating Out of SSA Form

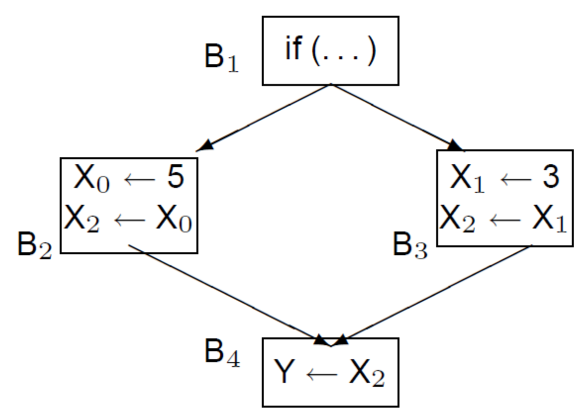
Overview:

1. Dead-code elimination (prune dead φs)
2. Replace φ-functions with copies in predecessors
3. Register allocation with copy coalescing

Before Step 2



After Step 2



Control Dependence

Def. Postdomination: node *p* postdominates a node *d* if all paths to the exit node of the graph starting at *d* must go through *p*

Def. In a CFG, **node Y is control-dependent on node B** if

- There is a non-empty path N0 = B,N1,N2, ...,Nk = Y such that Y postdominates N1 . . .Nk, and
- Y does not strictly postdominate B

Def. The Reverse Control Flow Graph (RCFG) of a CFG has the same nodes as CFG and has edge Y $\rightarrow$ X if X $\rightarrow$ Y is an edge in CFG.

- p is a postdominator of d iff p dominates d in the RCFG.

Computing Control Dependence

Key observation: Node Y is control-dependent on B *iff* $B \in DF(Y)$ in RCFG.

Algorithm:

1. Build RCFG
2. Build dominator tree for RCFG
3. Compute dominance frontiers for RCFG
4. Compute $CD(B) = \{Y \mid B \in DF(Y)\}$.

CD(B) gives the nodes that are control-dependent on B.

Summary

Complexity:

The conversion to SSA form is done in three steps:

(1) The *dominance frontier* mapping is constructed from the control flow graph *CFG* (Section 4.2). Let *CFG* have N nodes and E edges. Let *DF* be the mapping from nodes to their dominance frontiers. The time to compute the dominator tree and then the dominance frontiers in *CFG* is $O(E + \sum_X |DF(X)|)$.

(2) Using the dominance frontiers, the locations of the ϕ-functions for each variable in the original program are determined (Section 5.1). Let A_{tot} be the *total* number of assignments to variables in the resulting program, where each ordinary assignment statement $LHS \leftarrow RHS$ contributes the length of the tuple *LHS* to A_{tot}, and each ϕ-function contributes 1 to A_{tot}. Placing ϕ-functions contributes $O(A_{tot} \times avrgDF)$ to the overall time, where *avrgDF* is the weighted average (7) of the sizes $|DF(X)|$.

(3) The variables are renamed (Section 5.2). Let M_{tot} be the total number of mentions of variables in the resulting program. Renaming contributes $O(M_{tot})$ to the overall time.

Follow up works:

- A linear time algorithm for placing phi-nodes (POPL 1995) https://dl.acm.org/citation.cfm?id=199464
- Algorithms for computing the static single assignment form (JACM 2003)

Further reading:

- Tiger Book, Chapter 19
- On History: http://citi2.rice.edu/WS07/KennethZadeck.pdf