CONTROL FLOW ANALYSIS

The slides adapted from Vikram Adve

Flow Graphs

Flow Graph: A triple G=(N,A,s), where (N,A) is a (finite) directed graph, s ∈ N is a designated "initial" node, and there is a path from node s to every node n ∈ N.

- An *entry node* in a flow graph has no predecessors.
- An *exit node* in a flow graph has no successors.
- There is exactly one entry node, s. We can modify a general DAG to ensure this. *How?*
- In a control flow graph, any node unreachable from s can be safely deleted. *Why?*
- Control flow graphs are usually *sparse*. I.e., | A |= O(| N |). In fact, if only binary branching is allowed | A | ≤ 2 | N |.

Control Flow Graph (CFG)

Basic Block is a sequence of statements S_1 ... S_n such that execution control must reach S_1 before S_2, and, if S_1 is executed, then S_2 . . . S_n are all executed in that order

- Unless a statement causes the program to halt

Leader is the first statement of a basic block

Maximal Basic Block is a basic block with a maximum number of statements (n)

Control Flow Graph (CFG)

CFG is a directed graph in which:

- Each node is a single basic block
- There is an edge b1 $\rightarrow$ b2 if block b2 *may* be executed after block b1 in *some* execution

We define it typically for a single procedure

A CFG is a conservative approximation of the control flow! Why?

Example

Source Code

```
unsigned fib(unsigned n) {
   int i;
   int f0 = 0, f1 = 1, f2;

   if (n <= 1) return n;

   for (i = 2; i <= n; i++) {
      f2 = f0 + f1;
      f0 = f1;
      f1 = f2;
   }

   return f2;
}
```

LLVM bitcode

```
define i32 @fib(i32) {
  %2 = icmp ult i32 %0, 2
  br i1 %2, label %12, label %3

; <label>:3:
  br label %4

; <label>:4:
  %5 = phi i32 [ %8, %4 ], [ 1, %3 ]
  %6 = phi i32 [ %5, %4 ], [ 0, %3 ]
  %7 = phi i32 [ %9, %4 ], [ 2, %3 ]
  %8 = add i32 %5, %6
  %9 = add i32 %7, 1
  %10 = icmp ugt i32 %9, %0
  br i1 %10, label %11, label %4

; <label>:11:
  br label %12

; <label>:12:
  %13 = phi i32 [ %0, %1 ], [ %8, %11 ]
  ret i32 %13
}
```

Dominance in Flow Graphs

Let d, d1, d2, d3, n be nodes in G.

d **dominates** n ("d dom n") *iff* every path in G from s to n contains d

d **properly dominates** n if d dominates n and d $\neq$ n

d **is the immediate dominator of** n ("d idom n") if d is the last proper dominator on any path from initial node to n,

DOM(x**)** denotes the set of dominators of x.

Dominator Properties

Lemma 1: DOM(s) = { s }.

Lemma 2: s dom d, for all nodes d in G.

Lemma 3: The dominance relation on nodes in a flow graph is a *partial ordering*

- *Reflexive* — *n dom n* is true for all n.
- *Antisymmetric* — If *d dom n*, then not *n dom d*
- *Transitive* — $d1 \; dom \; d2 \wedge d2 \; dom \; d3 \Rightarrow d1 \; dom \; d3$

Lemma 4: The dominators of a node form a list.

Lemma 5: Every node except s has a unique immediate dominator.

Finding Dominators in a Flow Graph

Input : A flow graph G = (N,A,s).

Output : The sets DOM(node) for each node ∈ N.

```
DOM(s) := { s }

forall n ∈ N − {s} do
   DOM(n) := N
od

while changes to any DOM(n) occur do
   forall n in N − {s} do
      DOM(n) := {n} ∪ ⋂_{p→n} DOM(p)
   od
od
```

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od
```

Initialize

Iterate

Loops

```
while (b) { … }  ⇒   ?
```

Loops

The right definition of "loop" is not obvious.

Obviously bad definitions

- **Cycle:** Not necessarily properly nested or disjoint
- **Strongly Connected Components:** Too coarse; no nesting information

What properties of the loops do we want to extract from CFG?

Loops: Two Definitions

Natural loop — Defined using dominators

Intervals — Defined in terms of reachability in flow graph

Natural Loops

Def. **Back Edge:** An edge n → d where d dom n

Def. **Natural Loop:** Given a back edge, n → d, the natural loop corresponding to n → d is the set of nodes ***{d + all nodes that can reach n without going through d}***

Def. **Loop Header:** A node d that dominates all nodes in the loop

- Header is unique for each natural loop *Why?*
- Implies d is the unique entry point into the loop
- Uniqueness is very useful for many optimizations

Natural Loops

Pros:

+ Intuitive, and similar to SCC.
+ Single entry point: "loop header".
+ Identifies nested loops (if different headers)

Cons:

- Nested loops are not disjoint.
- Some nodes are not part of any natural loop.
- Does not include some cycles in "irreducible" flow graphs.

Reducibility of Flow Graphs

Def. **Reducible* flow graph**: a flow graph G is called reducible iff we can partition the edges into 2 disjoint sets:

- **forward edges**: should form a DAG in which every node is reachable from initial node s (or also header)
- **remaining edges must be back edges**: i.e., only those edges n $\rightarrow$ d such that *d dom n*

Idea:

Every "cycle" has at least one back edge

$\Rightarrow$ All "cycles" are natural loops

Otherwise graph is called irreducible.

* Well-structured

Loops: Two Definitions

Natural loop — Defined using dominators

Intervals — Defined in terms of reachability in flow graph

Interval Analysis*

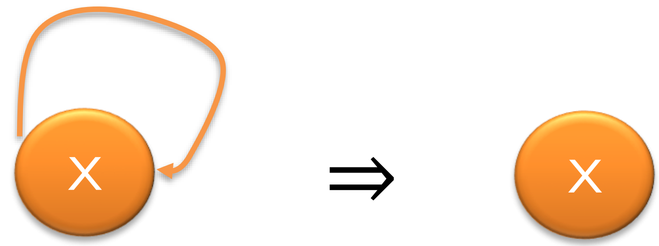
Idea: Partition flow graph into disjoint subgraphs so that each subgraph has a single entry (header).

Definition: The interval with node h as header, denoted I(h), is the subset of nodes of G constructed as:

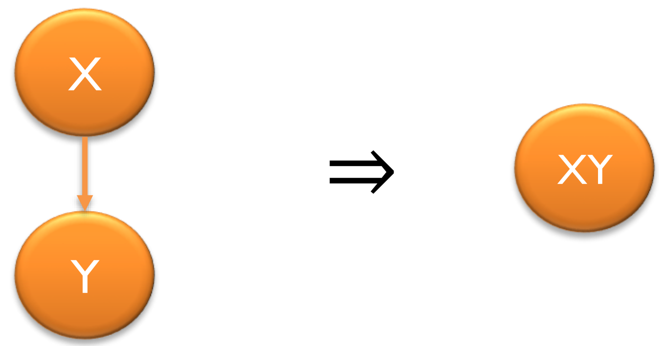
* It's different from the interval analysis on numerical quantities

Transformation Rules T1 and T2

T1 : Reduce a self-loop
$x \to x$ to a single node



T2 : If $x \to y$, and there is
no other predecessor of y,
then reduce x and y to a
single node.



Important: If G is reducible, successive applications of T1 and T2 produce the trivial graph.

⇒ Reducibility by T1 and T2 is equivalent to reducibility by intervals.

Node Splitting

Claim: If a node has n > 1 predecessors and m > 1 successors, split the node into n copies:

T2 is always applicable to a graph after a node is split

⇒ Any graph can be reduced to the trivial graph by applying T1, T2, and splitting.

Challenge: Finding a "minimal" splitting of a graph is not easy. Typically involves an NP-complete problem.

Interval Analysis*

Idea: Partition flow graph into disjoint subgraphs so that each subgraph has a single entry (header).

Definition: The interval with node h as header, denoted I(h), is the subset of nodes of G constructed as:

$I(h) := \{h\}$

while $\exists$ node m such that $m \notin I(h)$ and $m \neq s$ and
all arcs entering m leave nodes in $I(h)$

do

$I(h) := I(h) + m$

od

* It's different from the interval analysis on numerical quantities

Derived Flow Graphs

Def. **Derived Flow Graph, I(g)**: If G is a flow graph, then its I(G) is:

(a) The nodes of I(G) are the intervals of G

(b) The initial node of I(G) is I(s)

(c) There is an arc from node I(h) to I(k) in I(G) if there is any arc from a node in I(h) to node k in G.

Def. **Derived sequence**: the sequence $G = G_0, G_1, \ldots, G_k$ is derived *iff*

- $G_{i+1} = I(G_i)$ for $0 \leq i < k$,
- $G_{k-1} \neq G_k$,
- $I(G_k) = G_k$. G_k is called the limit flow graph of G.

Definition: A flow graph is reducible *iff* its limit flow graph is a single node with no arc. Otherwise it is called irreducible.

Intervals Properties

Lemma 6. I(h) is unique: does not depend on order of node insertion. (See *Hecht* for proof)

Lemma 7. The subgraph generated by I(h) is itself a flow graph.

Lemma 8.

(a) Every arc entering a node of the interval I(h) from the outside enters the header h.

(b) h dominates every node in I(h)

(c) every cycle in I(h) includes h

See You Next Time!

Review in the next few weeks:

Muchnick, Chapter 21: Case Studies of Compilers

Review by next class: Sections from Muchnick Sections §4.1-4.5, 4.9: Intermediate Representations

Section §7.1: Control Flow Graphs

(or equivalent sections in Cooper & Torczon or Aho, Lam, Sethi & Ullman)