CS 526

Advanced

Compiler

Construction

http://misailo.cs.Illinois.edu/courses/cs526

STATIC SINGLE ASSIGNMENT

The slides adapted from Vikram Adve

References

Cytron, Ferrante, Rosen, Wegman, and Zadeck, "Efficiently Computing Static Single Assignment Form and the Control Dependence Graph," *ACM Trans. on Programming Languages and Systems*, 13(4), Oct. 1991, pp. 451–490.

Muchnick, Section 8.11 (*partially covered*).

Engineering a Compiler, Section 5.4.2 (*partially covered*).

Definition of SSA Form

A program is in SSA form if:

- each variable is assigned a value in **exactly one** statement
- each **use** of a variable is **dominated** by the **definition**

Advantages of SSA Form

Makes def-use and use-def chains explicit:

These chains are foundation of many dataflow optimizations

- We will see some soon!

Compact, flow-sensitive* def-use information

- fewer def-use edges per variable: one per CFG edge

* Takes the order of statements into account

Advantages of SSA Form (cont.)

No anti- and output dependences on SSA variables

- Direct dependence: **A=1; B=A+1**
- Antidependence: A=1; **B=A+1;A=2**
- Output dependence: **A=1;A=2;** B=A+1

Cannot reoder

Explicit merging of values (φ): key additional information

Can serve as **IR for code transformations** (see LLVM)

Constructing SSA Form

Simple algorithm

1. insert φ-functions for every variable at every join
2. solve reaching definitions
3. rename each use to the def that reaches it (unique)

What's wrong with this approach?

1. too many φ-functions (precision)
2. too many φ-functions (space)
3. too many φ-functions (time)

Where do we place φ-functions?

```
V=...; U=...; W=...;
if (...) then {
     V = ...;
     if (...) {
        U = V + 1;
     } else {
        U = V + 2;
     }

     W = U + 1;
}
```

- For V?
- For U?
- For W?

Where do we place φ-functions?

```
V=...; U=...; W=...;
if (...) then {
      V1 = ...;
      if (...) {
          U1 = V1 + 1;
      } else {
          U2 = V1 + 2;
      }
      V2=φ(V1, V1);U3=φ(U1, U2);W1=φ(W0, W0)
      W1 = U3 + 1;
}
V3=φ(V0, V1); U4=φ(U0, U3); W2=φ(W0, W1)
```

- For V?
- For U?
- For W?

Intuition for SSA Construction

Informal Conditions

If a block X contains an assignment to a variable V, then a φ-function must be inserted in each block Z such that:

1. there is a non-empty path between X and Z,

2. there is a path from the entry block (s) to Z that does not go through X,

3. Z is the first node on the path from X that satisfies point 2.

Intuition for SSA Construction

Informal Conditions

If block X contains an assignment to a variable V, then a φ-function must be inserted in each block Z such that:

1. there is a non-empty path between X and Z, and

   the value of V computed in X reaches Z

2. there is a path from the entry block (s) to Z that does not go through X

   there is a path that does not go through X, so some other value of V reaches Z along that path(ignore bugs due to uses of uninitialized variables). So, two values must be merged at X with a φ

3. Z is the first node on the path from X to Z that satisfies point 2

   the φ for the value coming from X is placed in Z and not in some earlier node on the path

Intuition for SSA Construction

Informal Conditions

Iterating the Placement Conditions:

- After a φ is inserted at Z, the above process must be repeated for Z because the φ is effectively a new definition of V.
- For each block X and variable V, there must be at most one φ for V in X.

This means that the above iterative process can be done with a single worklist of nodes for each variable V, initialized to handle all original assignment nodes X simultaneously.

Minimal SSA

A program is in SSA form if:

- each variable is assigned a value in **exactly one** statement
- each **use** of a variable is **dominated** by the **definition** i.e., the use can refer to a unique name.

Minimal SSA: As few as possible φ-functions,

Pruned SSA: As few as possible φ-functions and no dead φ-functions (i.e., the defined variable is used later)

- One needs to compute liveness information
- More precise, but requires additional time

SSA Construction Algorithm

Steps:

1. Compute the dominance frontiers*
2. Insert φ-functions
3. Rename the variables

Thm. Any program can be put into minimal SSA form using the previous algorithm. [Refer to the paper for proof]

Dominance in Flow Graphs (review)

Let d, d1, d2, d3, n be nodes in G.

d **dominates** n ("d dom n") iff every path in G from s to n contains d

d **properly dominates n** ("d pdom n") if d dominates n and d $\neq$ n

d **is the immediate dominator of** n ("d idom n")
if d is the last proper dominator on any path from initial node to n,

DOM(x**)** denotes the set of dominators of x,

Dominator tree*: the children of each node d are the nodes n such that "d idom n" (d immediately dominates n)

Dominance Frontier

The dominance frontier of node X is the **set of nodes Y** such that **X dominates a predecessor of Y**, but X does not properly dominate Y *

DF(X) = {Y | ∃ P ∈ Pred(Y) : X dom P and not (X pdom Y)}

We can split DF(X) in two groups of sets:

$DF_{local}(X) \equiv \{Y \in Succ(X) \mid \text{not } X \text{ idom } Y\}$

$DF_{up}(Z) \equiv \{Y \in DF(Z) \mid \exists W.\ W \text{ idom } Z \text{ and not } W \text{ pdom } Y\}$

Then:

$$DF(X) = DF_{local}(X) \cup \bigcup_{Z \in Children(X)} DF_{up}(Z)$$

* **child, parent, ancestor, and descendant always refer to the dominator tree.**
predecessor, successor, and path always refer to CFG

Dominance Frontier Algorithm

for each X in a bottom-up traversal of the dominator tree (visit the node X in the tree after visiting its children):

```
DF(X) ← Ø

for each Y ∈ succ(X) /* local */
    if not X idom Y then
        DF(X) ← DF(X) U {Y}

for each Z ∈ children(X) /* up */
    for each Y ∈ DF(Z)
        if not X idom Y then
            DF(X) ← DF(X) U {Y}
```

Dominance and LLVM

LLVM mainline

Main Page | Related Pages | Modules | Namespaces | Classes | Files

File List | File Members

Dominators.h

Go to the documentation of this file.

```
//===- Dominators.h - Dominator Info Calculation ----------------*- C++ -*-===//
//
//                     The LLVM Compiler Infrastructure
//
// This file is distributed under the University of Illinois Open Source
// License. See LICENSE.TXT for details.
//
//===----------------------------------------------------------------------===//
//
// This file defines the DominatorTree class, which provides fast and efficient
// dominance queries.
//
//===----------------------------------------------------------------------===//

```

DominanceFrontier.h

Go to the documentation of this file.

```
//===- llvm/Analysis/DominanceFrontier.h - Dominator Frontiers --*- C++ -*-===//
//
//                     The LLVM Compiler Infrastructure
//
// This file is distributed under the University of Illinois Open Source
// License. See LICENSE.TXT for details.
//
//===----------------------------------------------------------------------===//
//
// This file defines the DominanceFrontier class, which calculate and holds the
// dominance frontier for a function.
//
// This should be considered deprecated, don't add any more uses of this data
// structure.
//
//===----------------------------------------------------------------------===//

#ifndef LLVM_ANALYSIS_DOMINANCEFRONTIER_H
#define LLVM_ANALYSIS_DOMINANCEFRONTIER_H

#include "llvm/IR/Dominators.h"
#include <map>
#include <set>

namespace llvm {

//===----------------------------------------------------------------------===//
/// DominanceFrontierBase - Common base class for computing forward and inverse
/// dominance frontiers for a function.
///
template <class BlockT>
class DominanceFrontierBase {
public:
  typedef std::set<BlockT *> DomSetType;                // Dom set for a bb
  typedef std::map<BlockT *, DomSetType> DomSetMapType; // Dom set map

protected:
  typedef GraphTraits<BlockT *> BlockTraits;

```

SSA Construction Algorithm

Steps:

1. Compute the dominance frontiers
2. Insert φ-functions
3. Rename the variables

Insert φ-functions

```
for each variable V
    HasAlready ← ∅
    EverOnWorkList ← ∅
    WorkList ← ∅
    for each node X that may modify V
        EverOnWorkList ← EverOnWorkList ⋃ {X}
        WorkList ← WorkList ⋃ {X}
```

Insert φ-functions

```
for each variable V
    HasAlready ← ∅
    EverOnWorkList ← ∅
    WorkList ← ∅
    for each node X that may modify V
        EverOnWorkList ← EverOnWorkList ⋃ {X}
        WorkList ← WorkList ⋃ {X}

    while WorkList ≠ ∅
        remove X from WorkList
        for each Y ∈ DF(X)
            if Y ∉ HasAlready then
                insert a ϕ-node for V at Y
                HasAlready ← HasAlready ⋃ {Y}
                if Y ∉ EverOnWorkList then
                    EverOnWorkList ← EverOnWorkList ⋃ {Y}
                    WorkList ← WorkList ⋃ {Y}
```

Renaming Variables*

Renaming definitions is easy – just keep the counter for each variable.

To **rename each use** of V :

(a) **Use in non-φ-functions**: Refer to immediately dominating definition of V (+ φ nodes inserted for V).

preorder on Dominator Tree!

(b) **Use as a φ-function operand**: Refer to the definition that immediately dominates the node with the incoming CFG edge (not the node with the φ-function)

rename the φ-operand when processing the predecessor basic block!

* For the full algorithm refer to the paper

```
j=1;

while (j < X)
    ++j;

N = j;
```

```
A: j = 1;

B: if (j >= X) goto E;

S:
j = j+1;
if (j < X) goto S;

E:
N = j;
```

```
j=1;

while (j < X)
    ++j;

N = j;
```

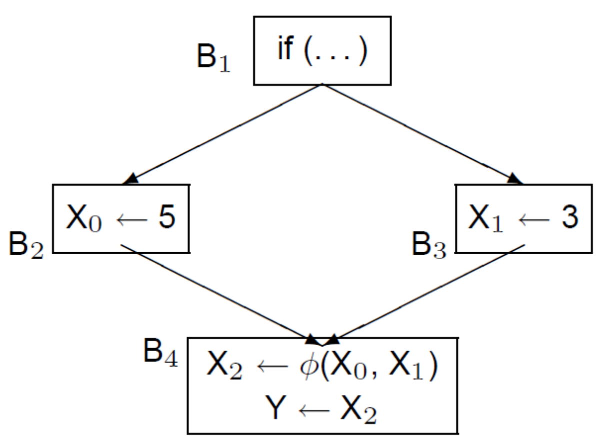
```
A: j0 = 1;

B: if (j0 >= X) goto E;

S: j1 = φ(j0, j2)
j2 = j1+1;
if (j2 < X) goto S;

E: j3 = φ(j0, j2)
N = j3;
```

Translating Out of SSA Form

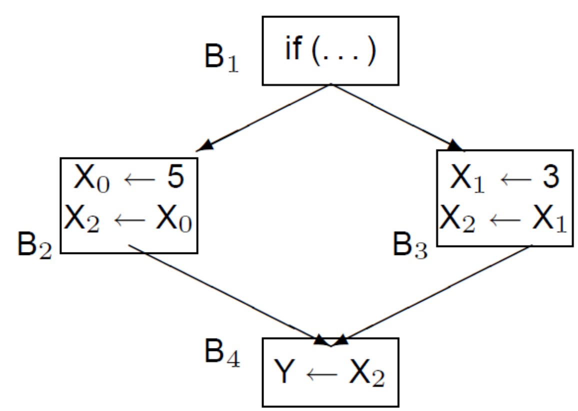
Overview:

1. Dead-code elimination (prune dead φs)
2. Replace φ-functions with copies in predecessors
3. Register allocation with copy coalescing

Before Step 2



After Step 2



Control Dependence

Def. Postdomination: node *p* postdominates a node *d* if all paths to the exit node of the graph starting at *d* must go through *p*

Def. In a CFG, **node Y is control-dependent on node B** if

- There is a non-empty path N0 = B,N1,N2, ...,Nk = Y such that Y postdominates N1 . . .Nk, and
- Y does not strictly postdominate B

Def. The Reverse Control Flow Graph (RCFG) of a CFG has the same nodes as CFG and has edge Y $\rightarrow$ X if X $\rightarrow$ Y is an edge in CFG.

- p is a postdominator of d iff p dominates d in the RCFG.

Computing Control Dependence

Key observation: Node Y is control-dependent on B *iff* $B \in DF(Y)$ in RCFG.

Algorithm:

1. Build RCFG
2. Build dominator tree for RCFG
3. Compute dominance frontiers for RCFG
4. Compute $CD(B) = \{Y \mid B \in DF(Y)\}$.

CD(B) gives the nodes that are control-dependent on B.

Summary

Complexity:

The conversion to SSA form is done in three steps:

(1) The *dominance frontier* mapping is constructed from the control flow graph *CFG* (Section 4.2). Let *CFG* have N nodes and E edges. Let DF be the mapping from nodes to their dominance frontiers. The time to compute the dominator tree and then the dominance frontiers in *CFG* is $O(E + \sum_X |DF(X)|)$.

(2) Using the dominance frontiers, the locations of the ϕ-functions for each variable in the original program are determined (Section 5.1). Let A_{tot} be the *total* number of assignments to variables in the resulting program, where each ordinary assignment statement $LHS \leftarrow RHS$ contributes the length of the tuple LHS to A_{tot}, and each ϕ-function contributes 1 to A_{tot}. Placing ϕ-functions contributes $O(A_{tot} \times avrgDF)$ to the overall time, where $avrgDF$ is the weighted average (7) of the sizes $|DF(X)|$.

(3) The variables are renamed (Section 5.2). Let M_{tot} be the total number of mentions of variables in the resulting program. Renaming contributes $O(M_{tot})$ to the overall time.

Follow up works:

- A linear time algorithm for placing phi-nodes (POPL 1995) https://dl.acm.org/citation.cfm?id=199464
- Algorithms for computing the static single assignment form (JACM 2003)

Further reading:

- Tiger Book, Chapter 19
- On History: http://citi2.rice.edu/WS07/KennethZadeck.pdf