Probabilistic & Approximate Computing

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In the previous episode…

TrueSkill: Model

Player skill: initially, we assume all players have similar (randomly assigned) skills, centered around some average:

$$Skill \sim Gaussian(100,10)$$

Player performance: it is based on the skill, but can be either higher or lower, depending on the moment of inspiration:

$$Perf \sim Gaussian(Skill, 15)$$

Tournament scores: Each player plays against each other, we can observe that a player with better performance won

$$PerfPlayerA > PerfPlayerB$$

TrueSkill Example

```
var trueSkill = function(){

  var skillA = gaussian(100, 10);
  var skillB = gaussian(100, 10);
  var skillC = gaussian(100, 10);

  var perfA1 = gaussian(skillA, 15), perfB1 = gaussian(skillB, 15);
  condition (perfA1 > perfB1);

  var perfB2 = gaussian(skillB, 15), perfC2 = gaussian(skillC, 15);
  condition (perfB2 > perfC2);

  var perfA3 = gaussian(skillA, 15), perfC3 = gaussian(skillC, 15);
  condition (perfA3 > perfC3);

  return skillA;
}
```

```
var res = Infer({method: 'MCMC', samples:
                     50000}, trueSkill)
print("Expected value: "+expectation(res));
viz.auto(res);
```

Inference with Continuous and Hybrid Models *(Exact and Approximate)*

Sampling (Rejection – Church)

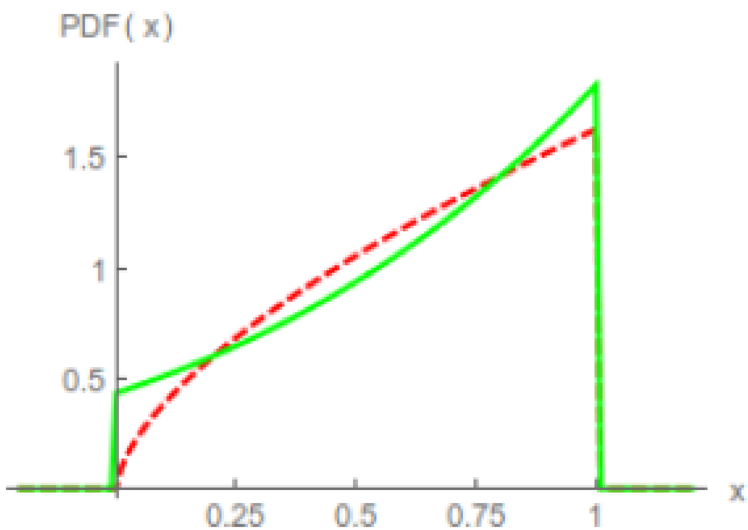
Sampling (MCMC – Church & Stan & Figaro)

Variational Inference (Fun & Infer.NET)

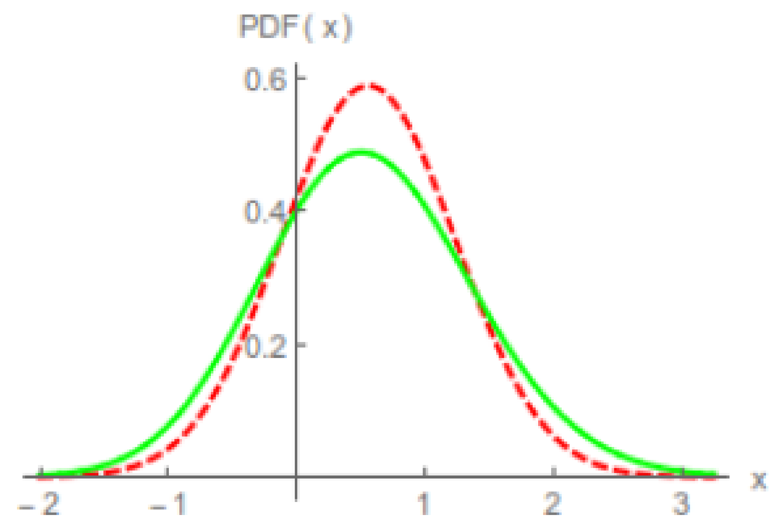
Exact Symbolic (PSI & Hakaru)

Approximate Inference

What queries to ask?



ClickGraph example



Maximum of two Gaussians

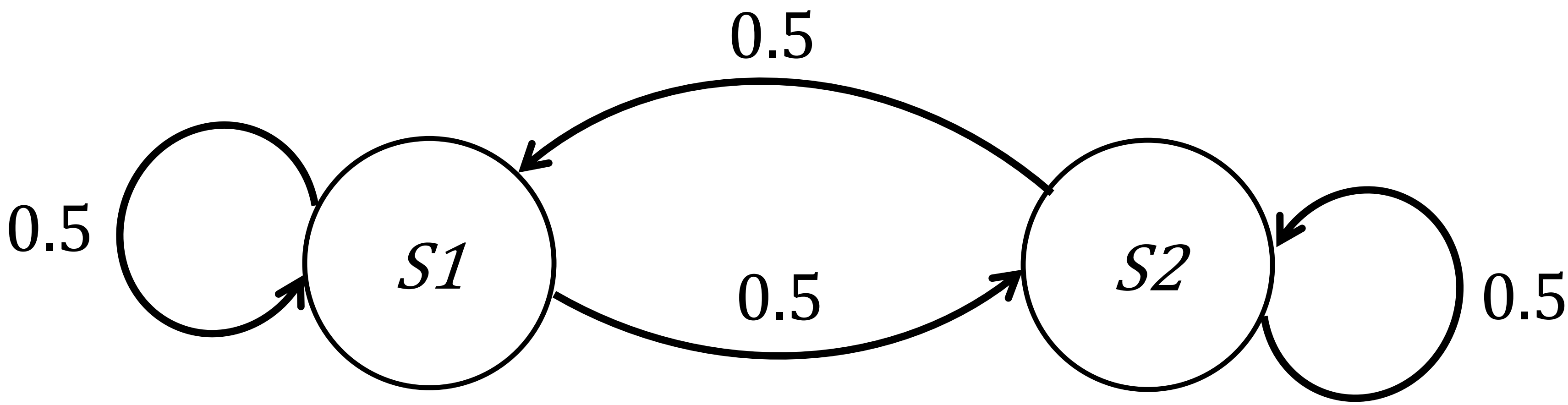
Comparison of PSI (exact inference) and Infer.NET (Variational approximate inference)
From "PSI: Exact Symbolic Inference for Probabilistic Programs", CAV 2016

Markov Chain *(discrete time)*

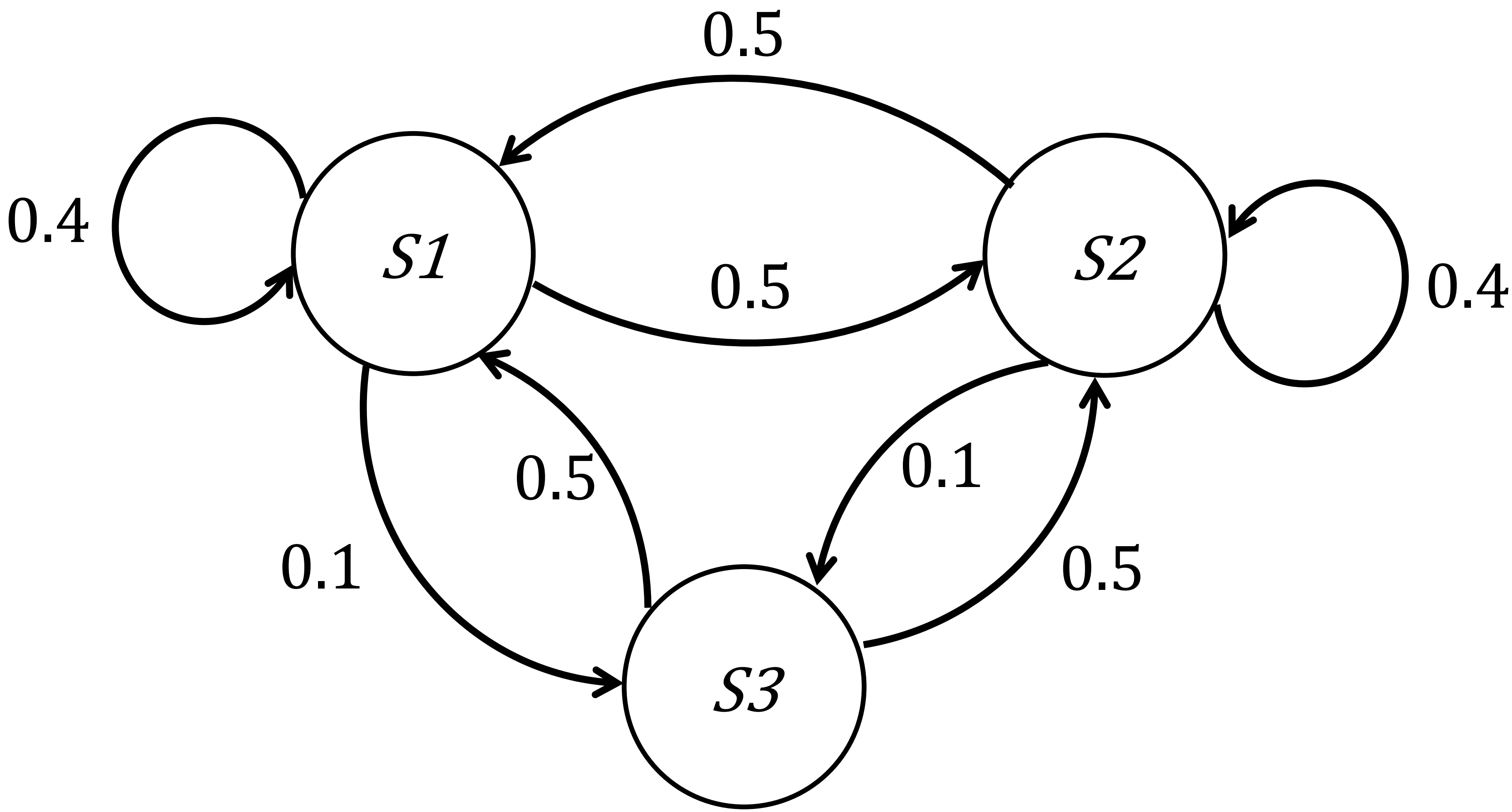
Graph (N, E, P, n_0) with the following elements:

- Nodes $n \in N$ represent states
- Directed edges $e \in E \subseteq N \times N$ are transitions from parent node to child node
- Transition probability function $P: E \to [0,1]$ labels each edge with a probability of transition.
  - For each node n, and outgoing edges $e' \in \{(n, n') \in E\}$ it holds that $\sum P(e') = 1$
- Starting node n_0

Markov Chain Example



Markov Chain Example



Markov Chain Properties

Distribution after k steps of iterating the chain:

$$\Pr(X_k \mid X_{k-1}, X_{k-2}, \dots X_0) = \Pr(X_k \mid X_{k-1})$$

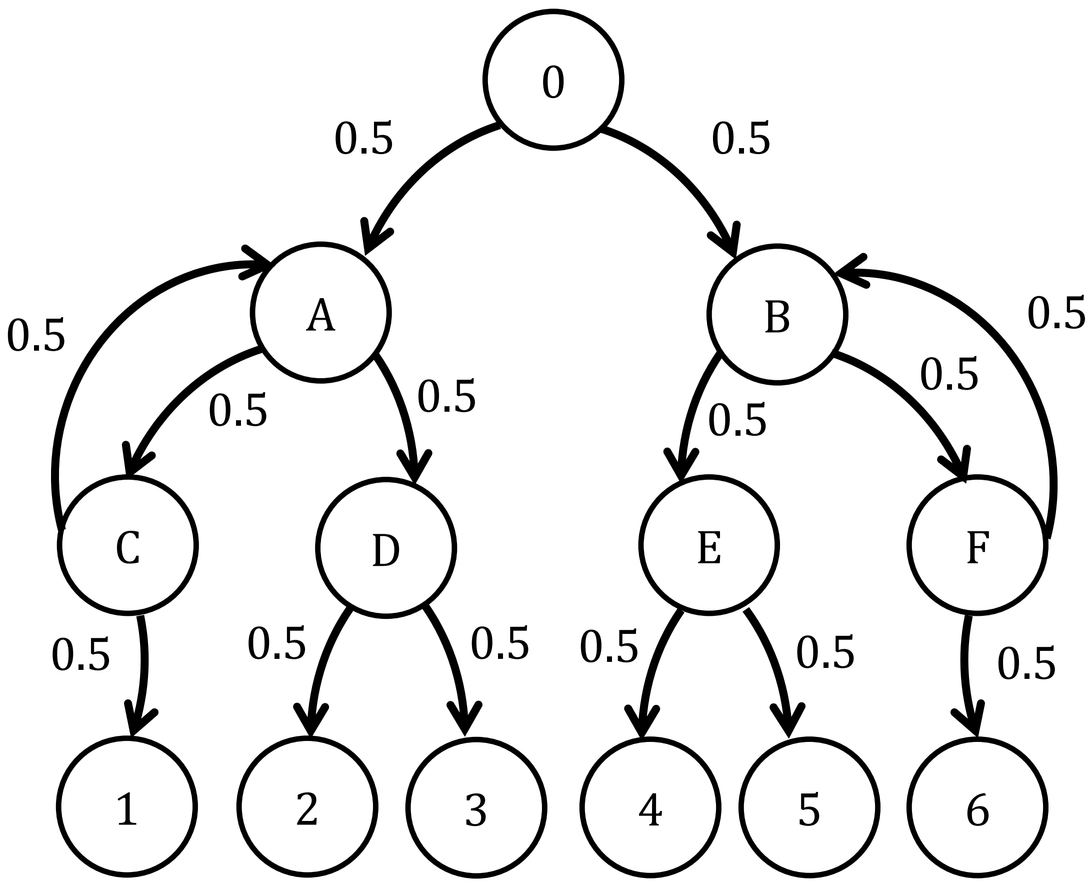
Markov property: the next transition depends only on the current state, not on the history.

Note, $\Pr(X_k = n' \mid X_{k-1} = n) = P((n, n'))$ – the transition probability between the edge (n, n').

The probability of a trace is then equal to the product of the transition probabilities.

Example: Simulating a Dice Roll

Implement a die roll using a fair coin (*)



* Example from "Probabilistic Programming", Gordon, Henzinger, Nori, Rajamani (ICSE-FoSE, 2014)

Example: Simulating a Dice Roll

```
var diceroll_iter = function(x){ // x is the current state

  if ( 1<=x && x<=6) return x;

  var coin = flip(0.5);
  var xnext =  // xnext is the next state
      x == 0x0? (coin? 0xA : 0xB) :
      x == 0xA? (coin? 0xC : 0xD) :
      x == 0xB? (coin? 0xE : 0xF) :
      x == 0xC? (coin? 0xA : 1):
      x == 0xD? (coin? 2: 3):
      x == 0xE? (coin? 4: 5):
      x == 0xF? (coin? 6: 0xB) : -100;

  return diceroll_iter(xnext);
}

var diceroll = function() { return diceroll_iter(0); }
```

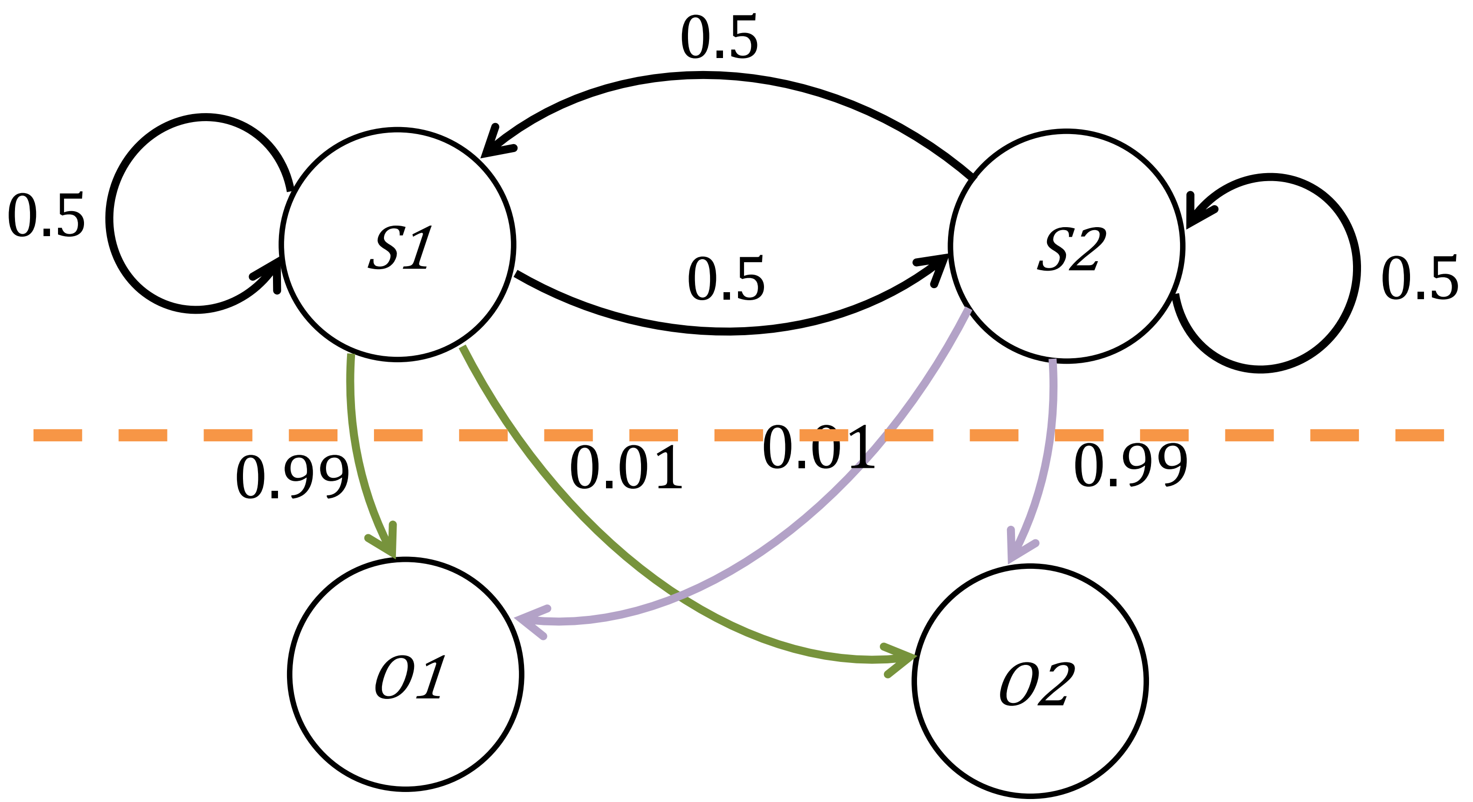
```
var res = Infer(
{method: 'enumerate',
maxExecutions: 24},
diceroll);
viz.auto(res);
```

Example: Simulating a Dice Roll

```
var diceroll_iter = function(x) { ... }

var diceroll = function() { ... }

var tworolls = function() {
 var r1 = diceroll();
 var r2 = diceroll();

 condition (r1 + r2 > 9)
 return r1;
}

var res = Infer({method: 'enumerate',
                 maxExecutions: 200},
                tworolls);
viz.auto(res);
```

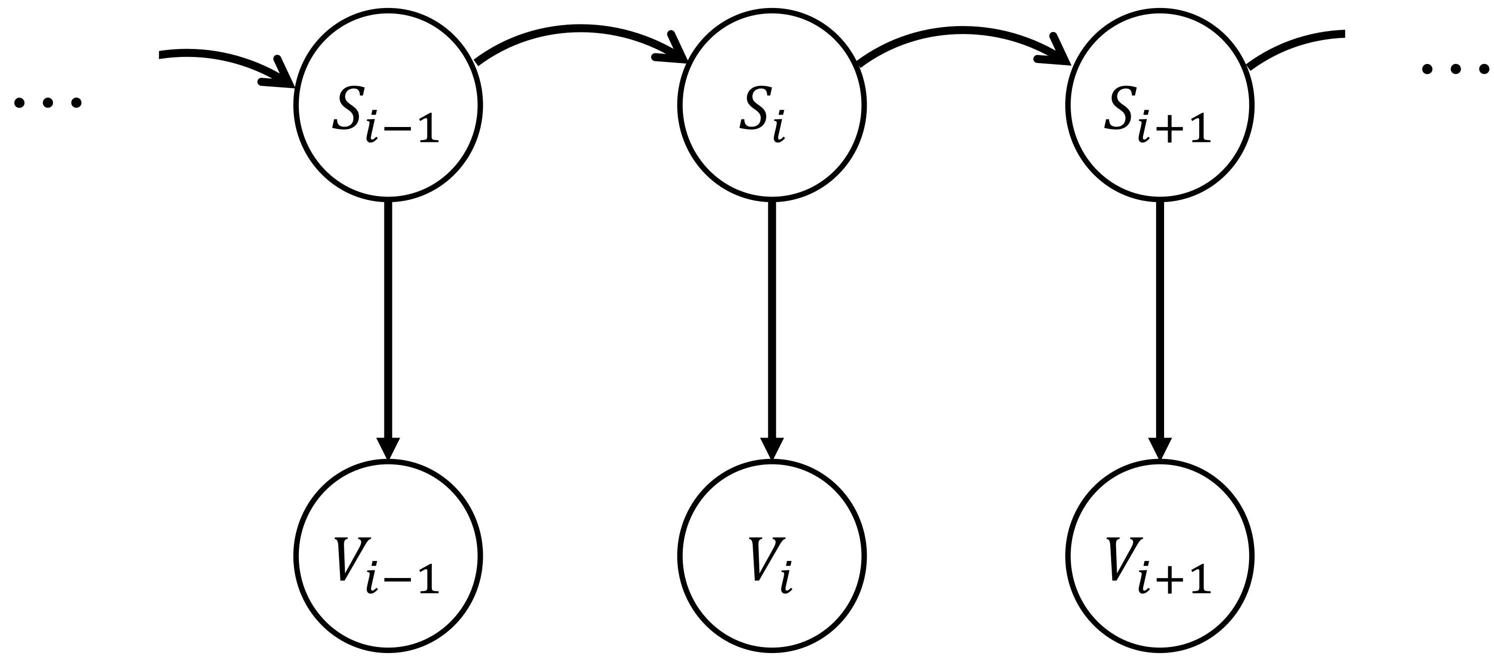
Hidden Markov Model

Has the base structure of a Markov model, but measurements may have noise



Hidden Markov Model

(Bayes Net representation)



Hidden Markov Model (Discrete)

```
var transition = function(s) {
  return s ? flip(0.7) : flip(0.3)
}

var measure = function(s) {
  return s ? flip(0.9) : flip(0.1)
  //return s  // <- This is just a Markov model
}

var hmm = function(n) {
  var prev = (n==1) ? {states: [], measurements:[]} : hmm(n-1)

  var newState = transition(prev.states[prev.states.length-1])
  var newObs = measure(newState)

  return {states: prev.states.concat([newState]),
          measurements: prev.measurements.concat([newObs])}
}

var trueObs = [false, true, false, true, false]

var model = function(){
  var r = hmm(trueObs.length)
  condition (_.isEqual(r.observations, trueObs))
  return r.states
};
```

For motivation and more details, see:
The Design and Implementation of Probabilistic Programming Languages,
Chapter 4 (Early, incremental evidence) http://dippl.org/chapters/04-factorseq.html