# **CS 598sm**

Probabilistic & Approximate Computing

http://misailo.web.engr.lllinois.edu/courses/cs598

# Nondeterministic Approximation in Parallel Computations

Removing synchronization and reading stale data Various techniques over the years:

- Dropping tasks (Rinard 2006 ICS)
- Removing barriers (Rinard 2007 OOPSLA)
- Reading stale data (Thies et al. PLDI 2011)
- Removing locks
- Parallelizing with data races (Misailovic et al. 2012, 2013)
- Breaking data dependencies

•

# Some Early Insights

```
iterate
{
  mask[1:M] = filter(...);
  parallel_iterate (i = 1 to M with mask[1:M] batch P)
  {
     ...
  }
} until converged(...);
```

Figure 4. Pseudocode of the best-effort iterative-convergence template.

We observe that the proposed iterative convergence template can be used to explore best-effort computing in three different ways.

- The selection of appropriate filtering criteria that reduce the computations performed in each iteration.
- The selection of convergence criteria that decide when the iterations can be terminated.
- The use of the batch operator to relax data dependencies in the body of the parallel\_iterate.

# Some Early Insights

```
iterate
{
  mask[1:M] = filter(...);
  parallel_iterate (i = 1 to M with mask[1:M] batch P)
  {
    ...
  }
} until converged(...);
```

Figure 4. Pseudocode of the best-effort iterative-convergence template.

Convergence-based pruning: Use converging data structures to speculatively identify computations that have minimal impact on results and eliminate them

**Staged Computation:** consider fewer points in early stages; gradually use more points in later stages to improve accuracy

**Early Termination:** Aggregate statistics to estimate accuracy and terminate before full convergence.

**Sampling:** Select a random subset of input data and use it to compute the results.

**Dependency Relaxation:** Ignore potentially redundant dependencies across iterations. Leads to more degree of parallelism or coarser granularity

# **Data Dependence**

A data dependence from statement \$1 to statement \$2 exists if

- I. there is a feasible execution path from SI to S2, and
- 2. an instance of S1 references the same memory location as an instance of S2 in some execution of the program, and
- 3. at least one of the references is a store.

# **Kinds of Data Dependence**

**Direct Dependence** 

**Anti-dependence** 

**Output Dependence** 

# Dependence Graph

#### A dependence graph is a graph with:

- Each node represents a statement, and
- Each directed edge from SI to S2, if there is a data dependence between SI and S2 (where the instance of S2 follows the instance of SI in the relevant execution).
  - SI is known as a source node
  - S2 is known as a sink node

# **Kinds of Data Dependence**

#### **Direct Dependence**

$$S2: \dots = X + \dots$$

# **Dependence Graph Edges**

$$S_1 \longrightarrow S_2$$

$$S_1 \longrightarrow S_2$$

#### Output Dependence SI:X = ...

$$S2: X = \dots$$

$$S_1 \longrightarrow S_2$$

# Dependence Graph for Loops

(Repeat) A dependence graph is a graph with:

- one node per statement, and
- a directed edge from SI to S2 if there is a data dependence between SI and S2 (where the instance of S2 follows the instance of SI in the relevant execution).

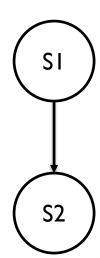
For loops: dependence graph is a summary of unrolled dependencies for different iterations

• Some (detailed) information may be lost

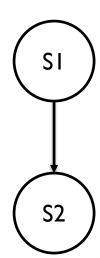
```
def X(), Y(), a(), i;
do i = 1 to N

S1:     X(i) = a(i) + 2

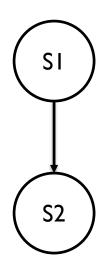
S2:     Y(i) = X(i) + 1
    enddo
```



```
def X(), Y(), a(), i;
do i = 1 to N
S1:     X(i+1) = a(i) + 2
S2:     Y(i) = X(i) + 1
enddo
```



```
def X(), Y(), a(), i;
do i = 2 to N
S1:     X(i) = a(i) + 2
S2:     Y(i) = X(i-1) + 1
enddo
```

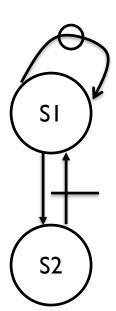


```
def X(), Y(), a(), i;
do i = 1 to N
S1:     X(i) = a(i) + 2
S2:     Y(i) = X(i+1) + 1
enddo
```

```
def X(), Y(), a(), i, t;
  do i = 1 to N

S1:     t = a(i) + 2

S2:     Y(i) = t + 1
  enddo
```



```
def X(), Y(), a(), i, t();
do i = 1 to N
S1:    t(i) = a(i) + 2
S2:    Y(i) = t(i) + 1
enddo
```

# STOCHASTIC GRADIENT DESCENT (SGD)



### MACHINE LEARNING AS OPTIMIZATION PROBLEM

minimize<sub>\theta</sub> 
$$L(\theta) = \mathbb{E}_{x \sim D} \ell(x, f(x, \theta))$$

- $\theta \in \mathbb{R}^P$  : model parameter
- D: data distribution
- $x \in \mathbb{R}^n$  : data sample
- ${\color{red} \bullet} \, f(\cdot, \cdot)$  : the model output given input and parameters
- $\ell(\cdot, \cdot)$  : loss function;

it's smaller, closer  $f(x,\theta)$  it gets to the ground truth  $m{e}$ 



## FINITE DATASET

- Usually, the dataset is finite.
- ullet Suppose there are N data samples, then it becomes

minimize<sub>\theta</sub> 
$$L(\theta) = \frac{1}{N} \sum_{i=1}^{N} \ell(x_i, f(x_i, \theta))$$



# 0123456789 0123456789 0123456789 0123456789 0123456789

**MNIST Classification** 

#### EXAMPLE

- Each sample  $x_i$  is given a true label  $y_i \in \{0, \dots, 9\}$ .
- Model outputs 10-dimension confidence vector in [0,1]<sup>10</sup> summing up to 1.
- The cross-entropy loss on the sample:

$$\ell(x_i, f(x_i, \theta)) = -\sum_{k=1}^{C} \mathbf{1}[k = y_i] \log(f(x_i, \theta))_k$$
$$= \log(1/f(x_i, \theta))_{y_i}$$

Smaller loss, higher confidence on the correct label, and higher accuracy.



# SGD

$$L(\theta) = \frac{1}{N} \sum_{i=1}^{N} \ell(x_i, f(x_i, \theta))$$

- A common way to solve the problem, is by using SGD:
  - Take the gradient of L with respect to  $\theta$ :  $\nabla_{\theta}L(\theta) (\in \mathbb{R}^{P})$
  - To minimize L, we move the  $\theta$  along the **opposite** direction:

$$\theta \leftarrow \theta - \gamma \nabla_{\theta} L(\theta)$$

- $\gamma$ : step size, a constant, positive small number
- Take sufficient such small steps, until  $L(\theta)$  does not change much.



# EXAMPLE

- In our MNIST task,  $f(x_i, \theta)_{y_i}$  is model **confidence score** for correct label
- Loss function:  $\ell(x_i, f(x_i, \theta)) = \log(1/f(x_i, \theta)_{y_i})$
- Gradient:  $\nabla_{\theta}\ell(x_i, f(x_i, \theta)) = -\frac{\nabla_{\theta}f(x_i, \theta)_{y_i}}{f(x_i, \theta)_{y_i}}$
- Parameter update by SGD:  $\delta = \gamma \frac{\nabla_{\theta} f(x_i, \theta)_{y_i}}{f(x_i, \theta)_{y_i}}$
- $\delta$ : model parameter change
- Direction: move towards larger confidence;
  - smaller confidence, sharper change.



# LEADING TO HOGWILD: ALGORITHM



# DECOMPOSE $\nabla_{\theta}L(\theta)$

$$L(\theta) = \frac{1}{N} \sum_{i=1}^{N} \ell(x_i, f(x_i, \theta))$$
$$\nabla L(\theta) = \frac{1}{N} \sum_{i=1}^{N} \nabla_{\theta} \ell(x_i, f(x_i, \theta))$$

A serial algorithm:

```
while (!converged(\theta))
for (int i=0; i<N; ++i)
\theta = \theta - 1/N * \nabla_{\theta} \ell(x_i, f(x_i, \theta))
```



# DECOMPOSE $\nabla_{\theta}L(\theta)$

$$L(\theta) = \frac{1}{N} \sum_{i=1}^{N} \ell(x_i, f(x_i, \theta))$$
$$\nabla L(\theta) = \frac{1}{N} \sum_{i=1}^{N} \nabla_{\theta} \ell(x_i, f(x_i, \theta))$$

A serial algorithm:

```
while (!converged(\theta))
for (int i=0; i<N; ++i)
\theta^{(t)} = \theta^{(t-1)} - 1/N * \nabla_{\theta} \ell \left(x_i, f(x_i, \theta^{(t-1)})\right)
```



# PARALLELISM?

A serial algorithm

```
for (int i=0; i<N; ++i)
for (int j=0; j<|\theta|; ++j)
\theta_j = \theta_j^{old} - 1/N * \nabla_{\theta_j} \ell\left(x_i, f(x_i, \theta^{old})\right)
```

One way to Parallelize

```
#parallel across K threads: for (int i=k*N/K; i < (k+1)* N/K; ++i) for (int j=0; j<|\theta|; ++j) \theta_j = \theta_j^{old} - 1/N * \nabla_{\theta_j} \ell\left(x_i, f\left(x_i, \theta^{old}\right)\right)
```



•Inner loop:

for (int j=0; j<
$$|\theta|$$
; ++j)
$$G(\theta_j^{old}) = \cdots$$

$$\theta_j = \theta_j^{old} - G(\theta_j^{old})$$

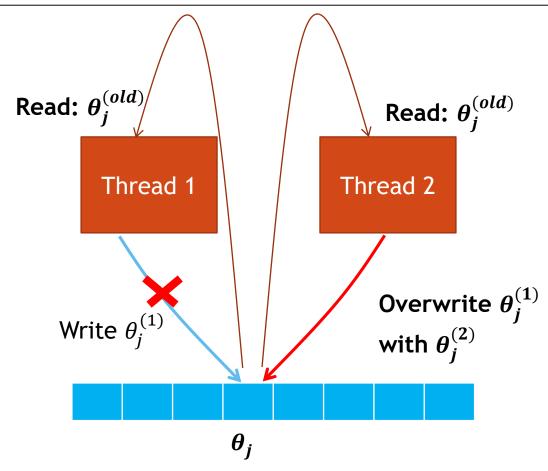
With some transformation:

for (int j=0; j<
$$|\theta|$$
; ++j)
$$G(\theta_j^{old}) = \cdots$$
if  $G(\theta_j^{old})! = 0$ 

$$\theta_j^{new} = \theta_j^{old} - G(\theta_j^{old})$$

#### For each sample:

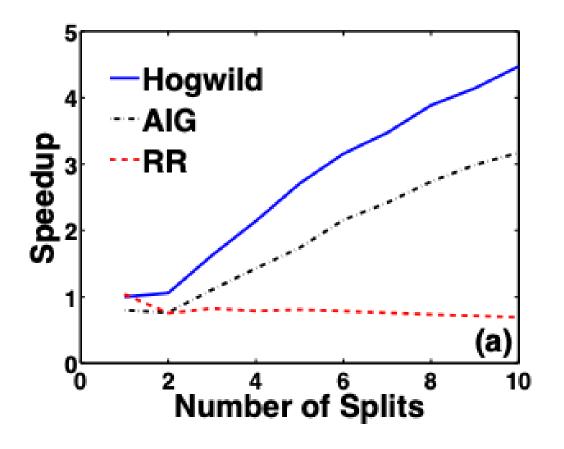
- •Only small number of parameters updated;
- •These parameters rarely overlap.





# PARALLELISM?

- The version "RR" tries to improve on the locking cost by using a round-robin schedule of updates
- The version "AIG" does a fine locking of the elements of  $\theta$
- Most of the time, the change will be for individual element of  $\theta$ , be even fine-grained locking is expensive





# KEY OBSERVATION: SPARSE SEPARABILITY

- The updates, even with the overwrite may give a good 'delta' direction
- Potential threat: it may not give 'strong enough' direction indication

- For many real-world problems, the model:
  - Usually has large number of parameters.
  - Only uses a small fraction of parameters to predict each data sample.
  - Parameters used for predicting different samples rarely overlap.
  - Each parameter is not often used.



## EXAMPLES

- Sparse SVM:
  - Data vector  $x_i$ 's are sparse.
- Matrix Completion:
  - Learn large matrix M as the product of AB, from few cells  $M_{ij}$ 's.
- Graph Cuts
  - Partition graph nodes according to sparse similarity matrix.



## RESULT ALGORITHM

- Update without lock is totally practical!
- Hogwild algorithm:

#### Algorithm 1 HOGWILD! update for individual processors

- loop
- 2: Sample e uniformly at random from E
- 3: Read current state  $x_e$  and evaluate  $G_e(x_e)$
- 4: for  $v \in e$  do  $x_v \leftarrow x_v \gamma G_{ev}(x_e)$
- 5: end loop
- e is data sample  $x_v = \theta$ ,  $G_e(x_e)$  is gradient.
- no lock on shared parameters  $x_e$ , totally asynchronous.



# PERFORMANCE & EVALUATION



# ASSUMPTIONS

We assume Lipschitz continuous differentiability of f with Lipschitz constant L:

$$\|\nabla f(x') - \nabla f(x)\| \le L\|x' - x\|, \ \forall x', x \in X.$$
 (8)

We also assume f is strongly convex with modulus c. By this we mean that

$$f(x') \ge f(x) + (x' - x)^T \nabla f(x) + \frac{c}{2} ||x' - x||^2$$
, for all  $x', x \in X$ . (9)

When f is strongly convex, there exists a unique minimizer  $x_{\star}$  and we denote  $f_{\star} = f(x_{\star})$ . We additionally assume that there exists a constant M such that

$$||G_e(x_e)||_2 \le M$$
 almost surely for all  $x \in X$ . (10)

We assume throughout that  $\gamma c < 1$ . (Indeed, when  $\gamma c > 1$ , even the ordinary gradient descent algorithms will diverge.) Our main results are summarized by the following



## THEORETICAL GUARANTEE

- Condition:
  - Convex function;
  - Gradient magnitude is bounded;
  - Number of workers is less than  $n^{1/4}$ , n is number of parameters;
  - Fine-tuned step size.
- •After  $k \geq \Theta\left(\frac{\log(1/\epsilon)}{\epsilon}\right)$  steps,  $\mathbb{E}[f(x_k) f_*] \leq \epsilon$ .
- Serial SGD convergence rate:  $\Theta(1/\epsilon)$  .
- Hogwild can be further optimized to get the same rate.



## **EXPERIMENTS**

Baseline approaches:

- •RR: processors are ordered; each update the decision variable in order
- •AIG: only lock particular parameters when updating ( $\theta_i$ 's with gradients)
- Hogwild: no locking

#### Three applications:

 SVM (Sparse SVM), MC (Matrix Completion), Cuts (Graph Cuts)



					HOGWILD!			KOUND KOBIN		
type	data	size	ρ	$\Delta$	time	train	test	time	train	test
	set	(GB)			(s)	error	error	(s)	error	error
SVM	RCV1	0.9	0.44	1.0	9.5	0.297	0.339	61.8	0.297	0.339
мс	Netflix	1.5	2.5e-3	2.3e-3	301.0	0.754	0.928	2569.1	0.754	0.927
	KDD	3.9	3.0e-3	1.8e-3	877.5	19.5	22.6	7139.0	19.5	22.6
	Jumbo	30	2.6e-7	1.4e-7	9453.5	0.031	0.013	N/A	N/A	N/A
Cuts	DBLife	3e-3	8.6e-3	4.3e-3	230.0	10.6	N/A	413.5	10.5	N/A
	Abdomen	18	9.2e-4	9.2e-4	1181.4	3.99	N/A	7467.25	3.99	N/A

Hogwarn

DOLLNID DODLNI

- Speed: Much faster than ordered locked update.
  - 9.5s vs 61.8s; 301.0s vs 2569.1s
- Accuracy: Almost the same training & test error.



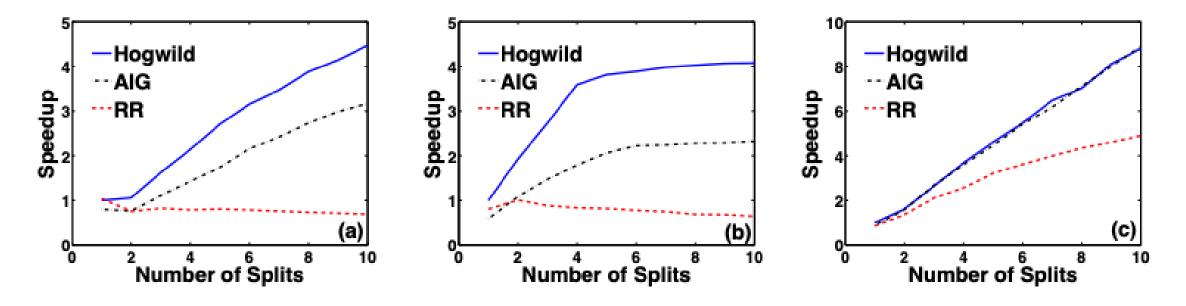


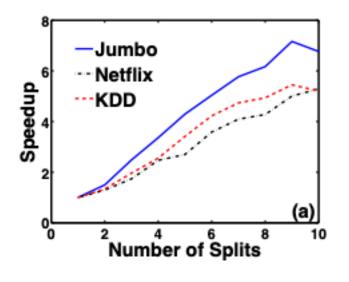
Figure 2: Total CPU time versus number of threads for (a) RCV1, (b) Abdomen, and (c) DBLife.

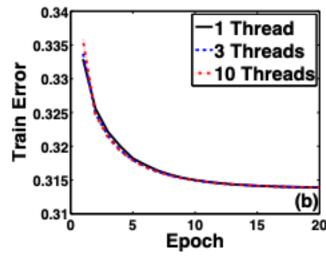
# SPARSE SVM PROBLEM WITH 3 DATASETS

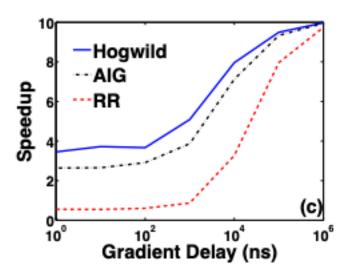
- Hogwild is much faster.
- Even only adding locks to all parameters, may significantly slow it down.



### MATRIX COMPLETION PROBLEM



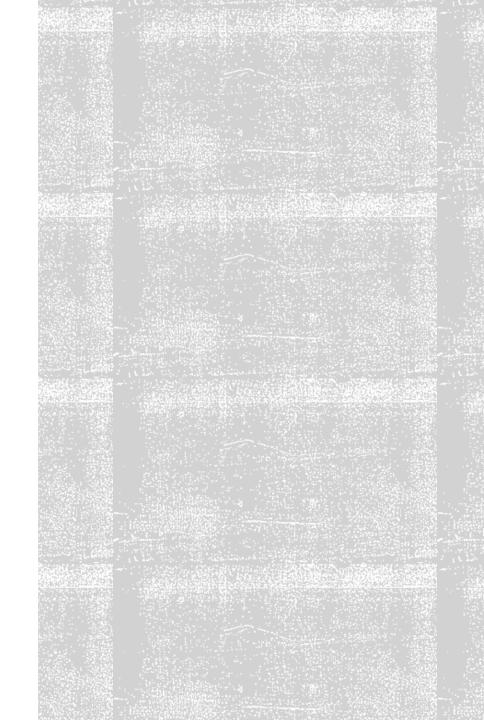




- Same trends for different datasets.
- Does not hurt accuracy.
- When gradient computation becomes slow, the gap shrinks.



# GENERALIZATION & RECENT PROGRESS



### HOW ABOUT NEURAL NETWORKS?

- The paper released in 2011, NN was not popular.
- SGD is also popular for NN training
- •NN is non-convex, no theoretical guarantee.

• Can Hogwild generalize to NN?



### IN TENSORFLOW

- Originally designed to use Hogwild (named asynchronous parameter updates).
- Also supports synchronous and synchronous with backups.
- See Tensorflow paper OSDI 2016

• In 2016, "Revisiting Distributed Synchronous SGD" (ICLR 2016 Workshop) experimented with comparing the strategies.



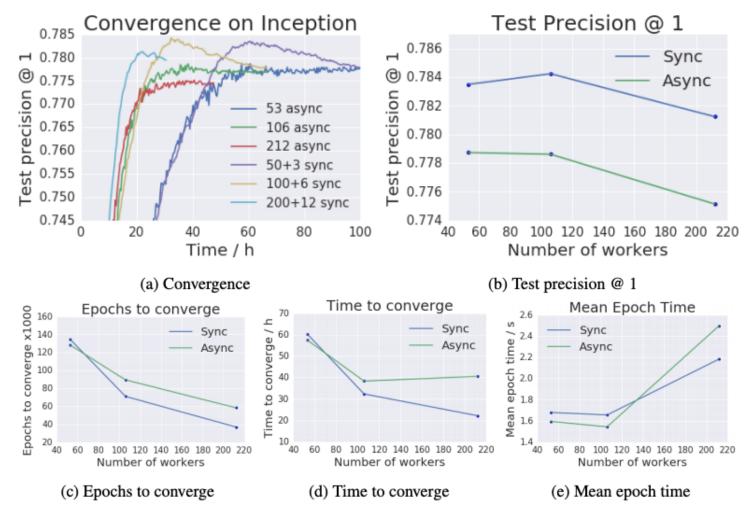


Figure 8: Convergence of Sync-Opt and Async-Opt on Inception model using varying number of machines. Sync-Opt with backup workers converge faster, with fewer epochs, to higher test accuracies.

- Async: similar to Hogwild
- Sync: lock and update; optimized

- In Hogwild, though each step may be faster, but more steps to converge.
- Slightly hurts accuracy, and takes more time to converge.



### RECENT APPROACH

- Synchronous with backup workers:
  - •n workers, but each step only requires m < n workers' result to update.
  - Overcome stragglers.
  - SGD samples training data randomly;
    - > each worker processes different batch;
    - **≻**OK if ignored.



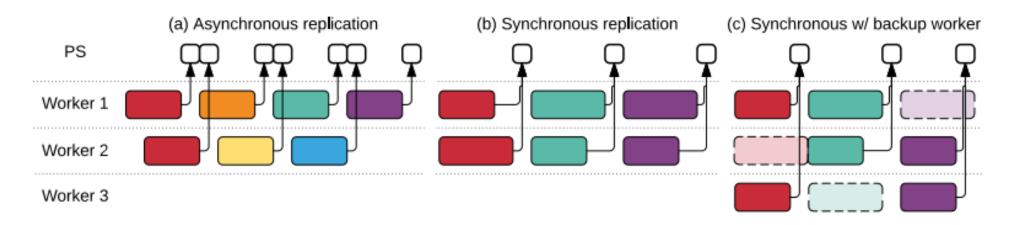
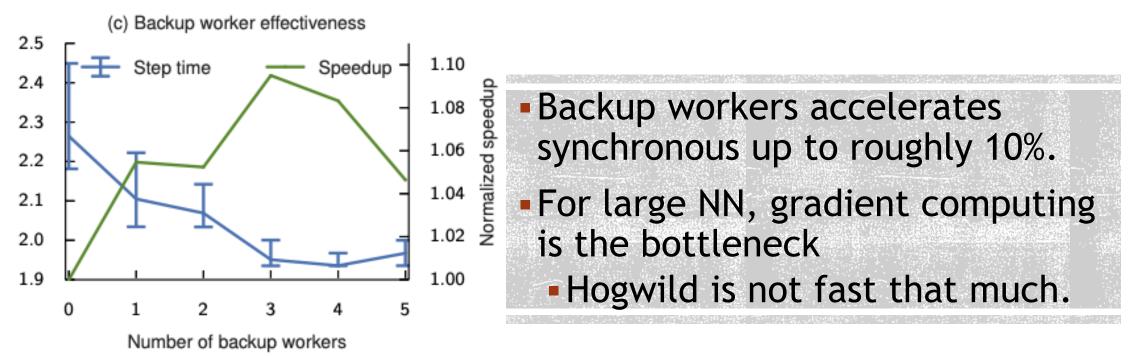


Figure 5: Three synchronization schemes for parallel SGD. Each color represents a different starting parameter value; a white square is a parameter update. In (c), a dashed rectangle represents a backup worker whose result is discarded.





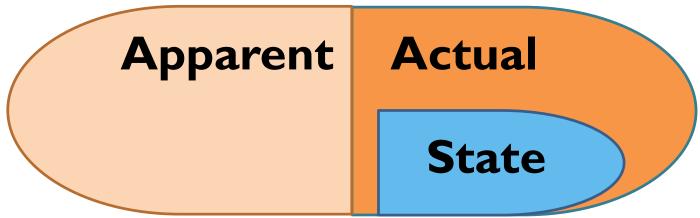
# LET'S THINK NOW ABOUT GENERAL PROGRAMS

- Removing synchronization and reading stale data
- Various techniques over the years:
- Dropping tasks (Rinard 2006 ICS)
- Removing barriers (Rinard 2007 OOPSLA)
- Reading stale data (Thies et al. PLDI 2011)
- Removing locks
- Parallelizing with data races (Misailovic et al. 2012, 2013)
- Breaking data dependencies

Studying various iterative and non-iterative programs, typical speedup is around 20% to 30%



# **Kinds of Dependencies**

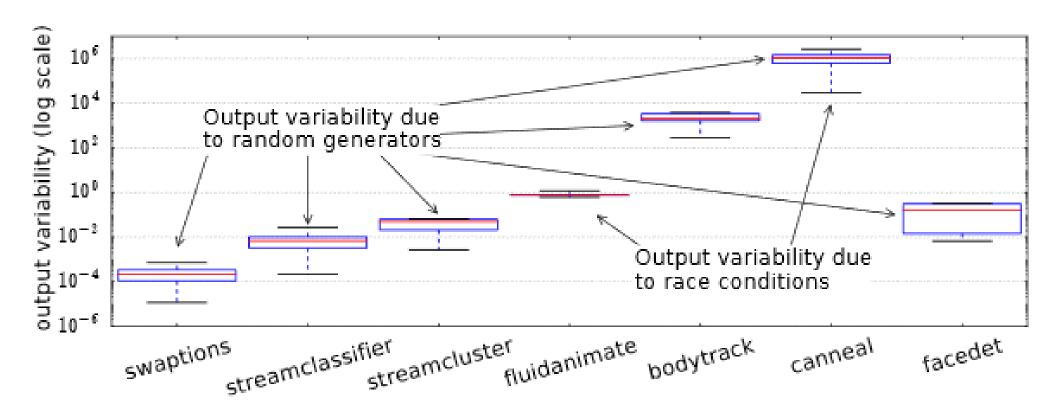


- Actual: exist in the program
- **State:** exist in the program and can be satisfied with extra code to match the original result, but faster than conventional
- Apparent: do not exist, but the compiler/developer cannot prove that they are unnecessary
- Strict preservation of every actual dependencies may not necessary, Preservation on any apparent dependency is not necessary

### Dependencies in Non-deterministic Codes?

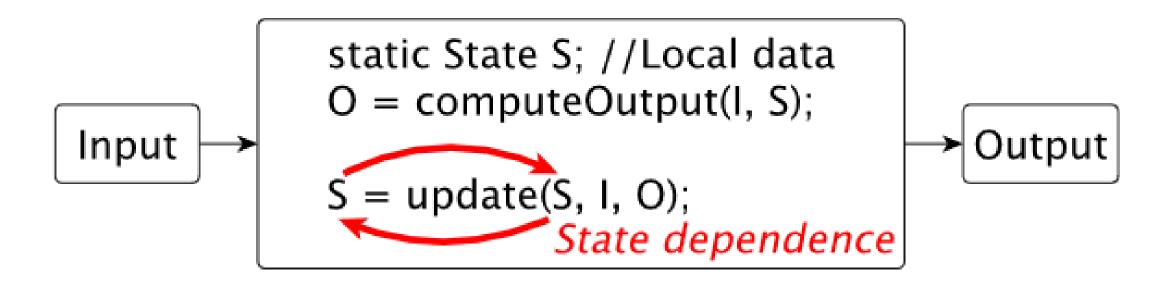
- For the same input, nondeterministic programs produce different results in each run.
- Use the error margins of the ordinary execution to find less important dependencies
- Non-determinism masks broken (unsatisfied) dependencies
- Use inexpensive checks to make sure the speculative execution matches those expected from the original program

### Opportunity for Accuracy (over 100 runs)



**Figure 2.** Output variability of nondeterministic PARSEC benchmarks. Several exhibit very high variability and are particularly amenable to STATS.

### **Opportunity State Dependency**

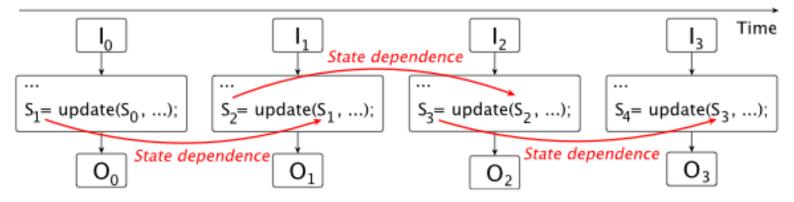


- Thread level parallelism is constrained by a sequential chain of dependences
- Opportunity: break this dependence to increase parallelism
- Fix: do 'speculation', if the result is too different, drop those updates and reexecute

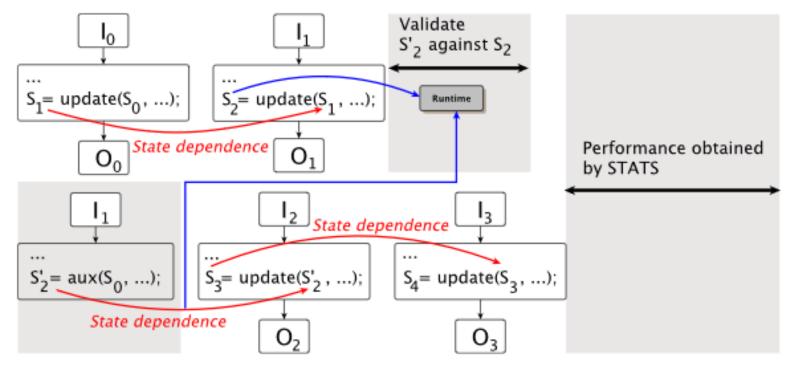
# Approach

Break the dependency occasionally

- Run inexpensive transfer function Ensure that the impact is not large
- If small, continue,
- If large, reexecute (infrequently)



(a) Execution serialization due to a state dependence



(b) Additional TLP generated by auxiliary code

### **Code Modification**

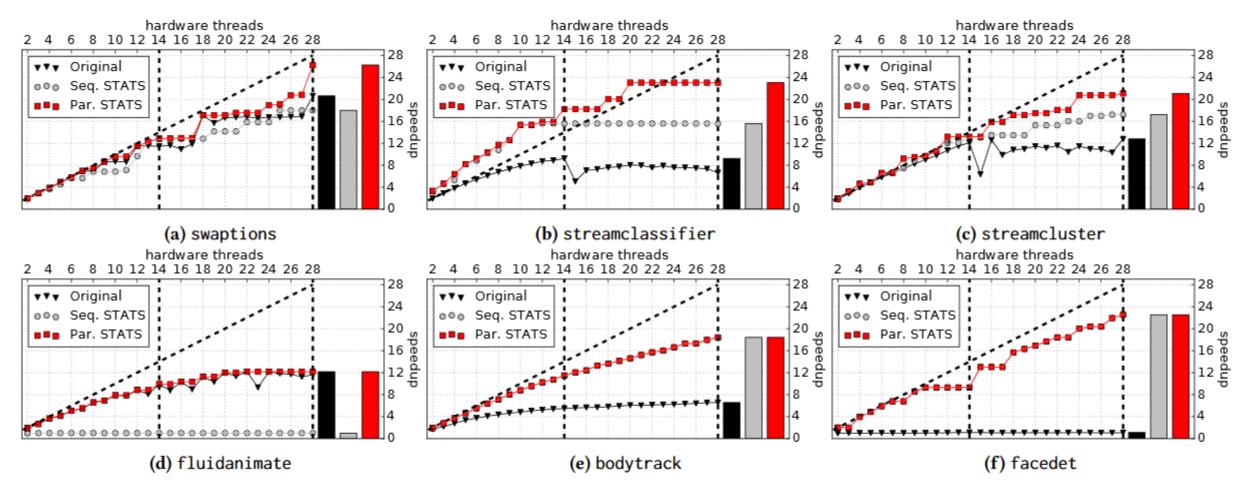
#### Bodytrack: Pose estimation program

**Figure 7.** Original code of bodytrack.

```
class Input { int frameId; };
class Output { vector < BodyPart > positions; };
class State {
                                                    3
  vector<Particle> model:
  State& operator = (State&);
  bool doesSpecStateMatchAny(set<State*>);
Output * computeOutput(Input *i, State *s){
  Frame f = getFrame(i->frameId);
  s->model = updateModel(TO_numAnnealingLayers,
                                                    10
                          s->model, f);
                                                    11
  Output *o = new Output();
                                                     12
  o->positions = getPositions(s->model);
                                                     13
  return o;
                                                     14
                                                     15
void estimateLocations() {
                                                     16
  vector < Input* > i(numFrames);
                                                    17
  vector<Particle> model(numParticles);
                                                     18
  State s; s.model = model;
                                                     19
  StateDependence < Input, State, Output >
                                                    20
                  stateDep(&i,&s,computeOutput);
                                                    21
  stateDep.start(); stateDep.join();
                                                     22
                                                     23
```

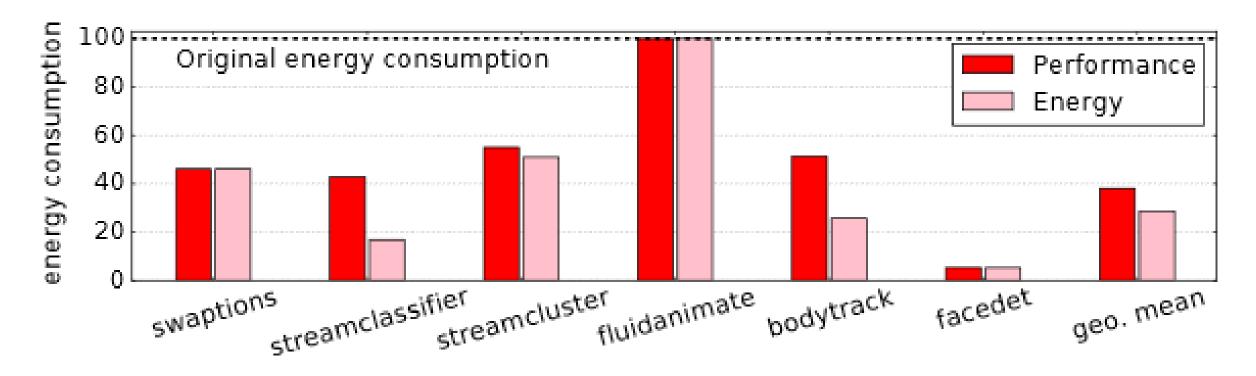
**Figure 8.** Use of SDI in bodytrack.

# **Extracting Parallelism: Speedup**



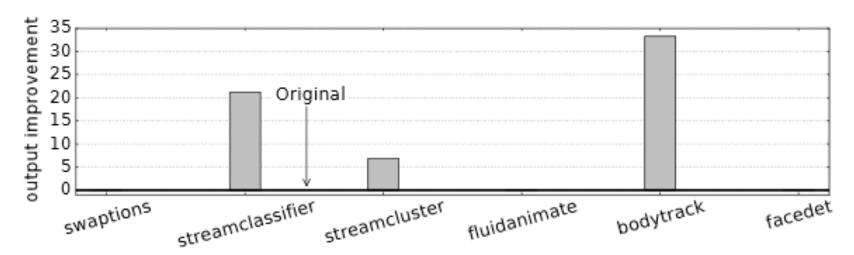
**Figure 12.** For most benchmarks, STATS generates a significant amount of extra parallelism that saturates the hardware resources of our platform. "Original" is the out-of-the-box benchmark that has been parallelized by traditional means. "Seq. STATS" ("Par. STATS") is the binary generated by STATS starting from the sequential (multi-threaded) version of a benchmark. The bar graphs show maximum speedup.

# **Energy Consumption**



**Figure 15.** The binaries generated by STATS use considerably less energy compared to the original benchmarks.

### **Accuracy Impact: Can run more**



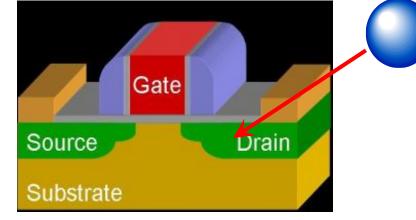
**Figure 16.** STATS can increase the original output quality by spending the saved time to iterate more over the same dataset.

Where is it good to use: Applications that analyze a long stream of data (e.g., bodytrack, facedet, streamcluster) where the information about inputs that is automatically computed (e.g., 3Dlocation of bodies, 2D location of faces, centroids of multi-dimensional points) has the "short memory" dependence property.

#### Soft Errors: Nondeterminism from Hardware

As technology scales, hardware reliability is more important

Hardware more susceptible to transient (soft) errors



**Soft Error** 

Many applications require very high reliability guarantees

TRANSPORTATION UBER RIDE-SHARING

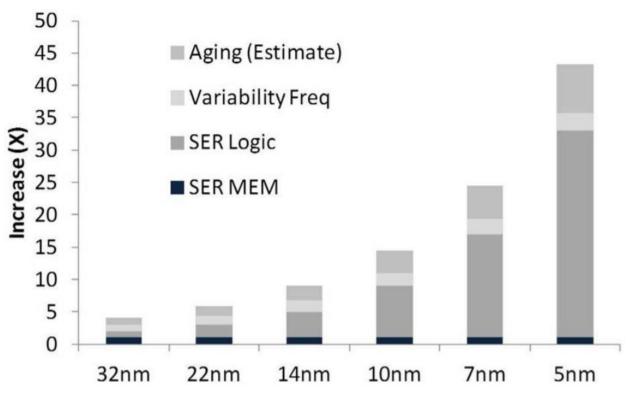
Uber self-driving car saw pedestrian but didn't brake before fatal crash, feds say

The report is more interesting for what it doesn't say than what it does

By Andrew J. Hawkins | @andyjayhawk | May 24, 2018, 11:07am EDT

"Volkswagen reported ~20% disengagements due to software hang/crashes", WAYMO, CA DMV 2016 Dataset, DSN 2018

### Unreliable Hardware



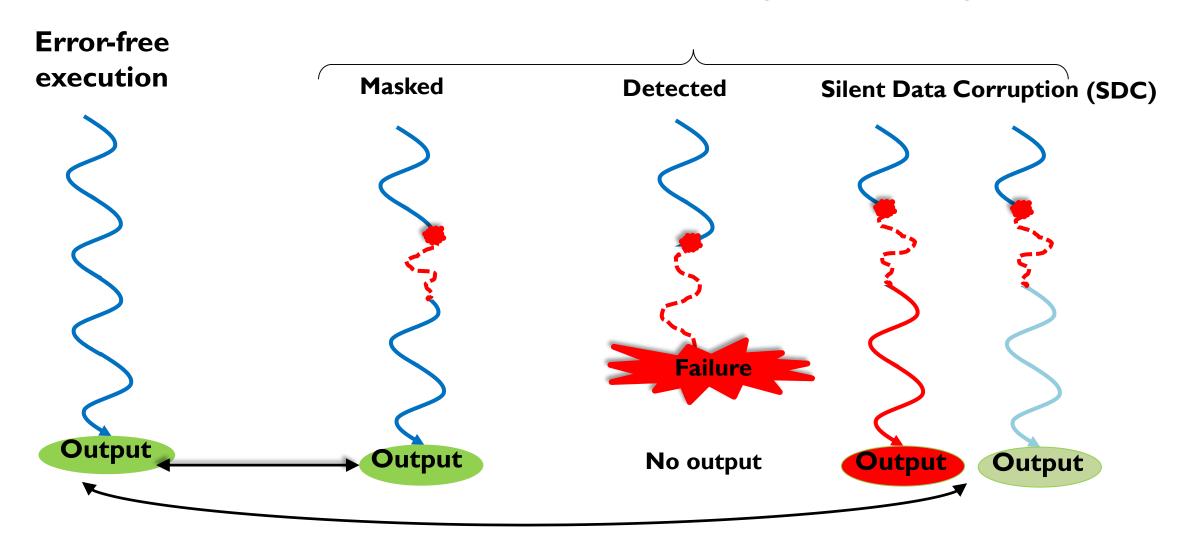
Architects make great efforts to minimize errors

Some errors slip through the cracks – silently corrupt computation results

Process size vs. error

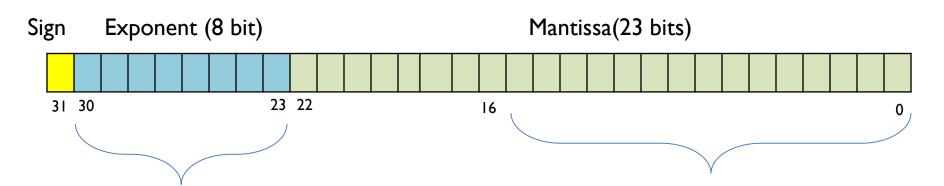
rate
Image from "Inter-Agency Workshop on HPC Resilience at Extreme Scale", DoD, '12

#### **Erroneous executions (has soft errors)**



### How do We See at Software Level?

float x:

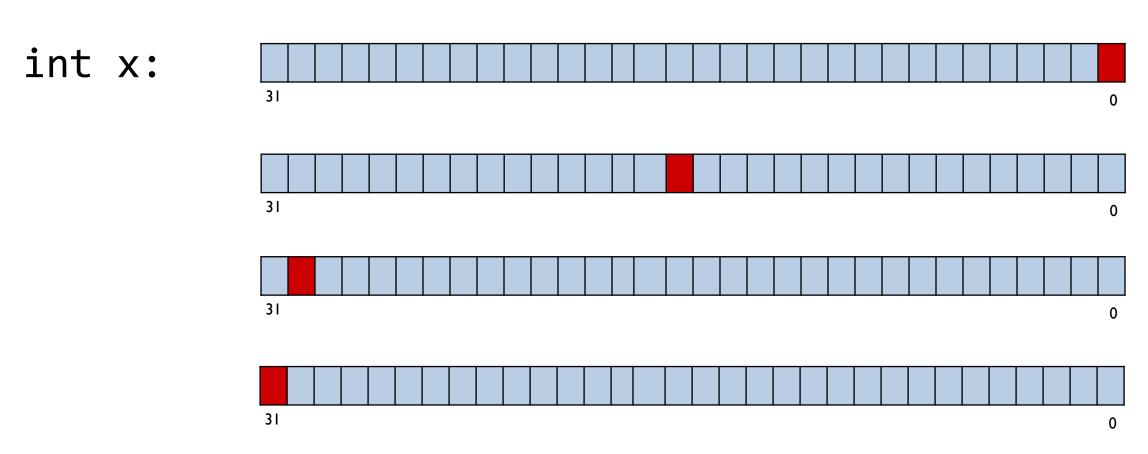


Often large impact

Often small impact

### How do We See at Software Level?

# **Corrupted Bits**



But also int\* x... what happens then?

# **Challenges and Traditional Solutions**

#### **Detection:**

- Run twice, compare the results
- Instruction Replication
- Algorithm-based fault tolerance

#### **Recovery:**

- Checkpoint-restart
- Run three times, do majority voting

#### **Detection:**

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Run exact and approximate versions, ensure they don't differ by too much

#### **Detection:**

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Replicate only some instructions

For the others, either rely on the property of the computation or develop inexpensive checkers

#### **Detection:**

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Make the algorithmic techniques aware of the approximation

#### **Detection:**

- Run twice, compare the results
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#### **Recovery:**

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Checkpoint only a small part of the state

Restart only when necessary

#### **Detection:**

- Run twice, compare the results
- Instruction Replication
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#### **Recovery:**

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- Run three times, do majority voting

If we need to re-execute, run only approximate algorithm

Try to do 'local repair' on the output

### Lightweight Check and Recover

```
z = x*y
z' = x*y
z==z'?
```

Code
Re-Execution
(SWIFT, DRIFT,
Shoestring)

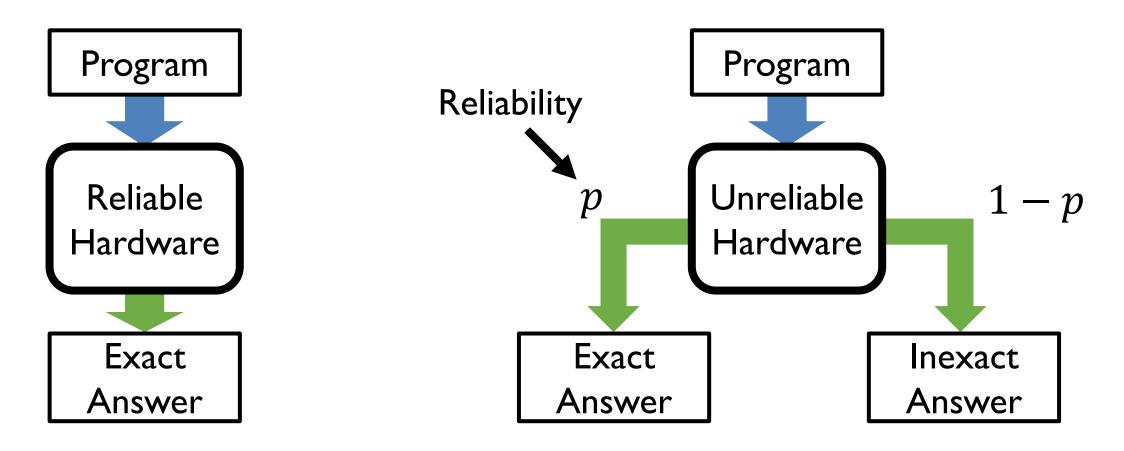
```
y = foo(x)
DNN(x,y)?
```

Anomaly
Detection
(Topaz, Rumba)

```
s = SAT(p)
verify(s,p) ?
```

Verification (for NP-Complete)

### Reliability



Reliability is the probability of obtaining the exact answer

# The Try-Check-Recover Mechanism

Some research languages 1,2 expose *Try-Check-Recover* mechanisms:

```
try { solution = SATSolve(problem) }

Checks for errors
check { satisfies(problem, solution) }

Recovery code
recover { solution = SATSolve(problem) }
```

<sup>1&</sup>quot;Relax", M. de Kruijf, S. Nomura, and K. Sankaralingam, ISCA '10

### Code Re-Execution – SWIFT<sup>I</sup>

```
// Instruction 1
try { z = x*y [p try] rnd(); }
check { z == (x*y [p try] rnd()) }
recover { z = x*y [p rec] rnd(); }
// Instruction 2
try { w = x+y [p try] rnd(); }
check { w == (x+y [p try] rnd()) }
recover { w = x+y [p rec] rnd(); }
```

<sup>&</sup>lt;sup>1</sup>G.A. Reis, J. Chang, N. Vachharajani, R. Rangan, and D. August, CGO '05

### Code Re-Execution – DRIFT<sup>I</sup>

```
// Instruction 1 and 2
try {
  z = x*y [p_try] rnd();
 w = x+y [p_try] rnd();
check {
 z == (x*y [p try] rnd()) && w == (x+y [p try] rnd())
recover {
 z = x*y [p_rec] rnd();
 w = x+y [p_rec] rnd();
```

# Code Re-Execution — Shoestring<sup>1</sup>

```
// Instruction 1
try { z = x*y [p_try] rnd(); }
check { z == (x*y [p_try] rnd()) }
recover { z = x*y [p_rec] rnd(); }
// Instruction 2 not considered critical
w = x+y [p_try] rnd();
```

# Anomaly Detection – Topaz<sup>1</sup>

```
try {
 z = f(x,y) [p_try] rnd();
check {
  isUnusual(x,y,z)
recover {
  z = f(x,y) [p rec] rnd();
```

### Hardware Error Flag<sup>1,2</sup>

```
try {
  z = x*y [p try] rnd();
check {
  !(read hw err_flag())
recover {
  z = x*y [p rec] rnd();
```